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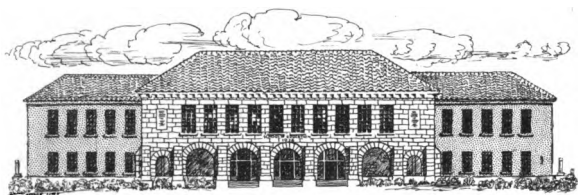
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PRACTICAL ARITHMETIC

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WARREN B. BROWN,

A

PRACTICAL ARITHMETIC,

DESIGNED FOR

GRAMMAR SCHOOLS AND ACADEMIES.

BY

GEO. E. SEYMOUR, A.M.,

Instructor in the St. Louis High School.

ST. LOUIS:

G. I. JONES AND COMPANY.

1880.

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P R E F A C E.

The Elementary Arithmetic and the Practical Arithmetic are designed to embrace all those subjects usually found in the more voluminous series of Arithmetics now in very general use. By a methodical arrangement of topics, all unnecessary repetition has been avoided; and by giving prominence to those subjects alone which are the key to the science of Arithmetic, brevity has been secured without sacrificing either fulness or clearness of treatment.

No attempt has been made to make the book either a complete dictionary of mathematical terms or a mere encyclopædia of useful mathematical knowledge. Arithmetic should be regarded in the school-room both as a science and as an art: as a science, so far as it deals with the principles which underlie its processes and its rules; and as an art, so far as it exemplifies these rules and these processes in their varied applications in solving problems that arise in the business of every-day life.

The complaint that Arithmetic is not successfully taught in the schools of the present day, is attempted to be met by the employment of a more rational method. The difficulties generally supposed to be inseparable from this study, arise partly from want of method and clearness in its treatment, partly from the attempt to carry along too many simultaneous studies. The time now allotted (hardly devoted) to this study is quite sufficient to guarantee a thorough mastery of its general principles and their application in practice; but the teaching of Arithmetic is still, to a great extent, merely an experiment. The prominence given this study in our schools for so many years, should by this time have relieved it from this charge. Whatever is radically wrong in our methods of teaching Arithmetic should be remedied. Whatever is radically wrong in the subject itself should be eradicated. The time allotted to this study is ample

for the end proposed. Method and thoroughness are the vital elements of success here as everywhere.

Fragmentary knowledge of any subject is of little value as an educational means, and too often quite useless as an end. Teachers who fail to give pupils a clear idea of the parts of a subject in their relation to the subject as a whole, generally leave them without that command over its details in practice, which a more thorough knowledge of fundamental principles would certainly give.

Unfortunately, the study of Arithmetic has suffered most from those who regard it, not as a *system* of principles and rules, but as a mere *aggregation* of rules to be mechanically applied in obtaining results. Nothing but intelligent and judicious effort can counteract this fatal tendency. Both in the Elementary and the Practical an effort is made to overcome this difficulty by referring all the operations to the fewest possible principles. While the practical applications of the rules of Arithmetic are manifold, the fundamental principles are few.

Since mental fibre is a growth it becomes refined and strengthened by proper exercise; hence, a pupil is benefitted more by what he does himself than by what others do for him. For this reason it is generally better to suggest a line of thought than to furnish him with a full argument; to give him suggestive solutions than to give him solutions in full, which seldom stimulate the mind of a pupil sufficiently to secure a thorough understanding of the logic of the solution. This feature of the book is commended to the consideration of teachers.

Special attention is called to the treatment of Percentage and its applications; to Compound Proportion as involving merely a series of simple proportions; to a new and exhaustive statement of the principles which underlie the subject of Equation of Accounts.

The power of Arithmetical Analysis is exemplified under that head by solutions given to a large variety of problems usually regarded as difficult. These solutions, together with the collection of a much larger number and variety of problems arranged under them than is found in any other book on this subject, will prove a valuable aid to many teachers who have Institute work to do.

In the key to this book additional examples and problems will be found, designed fully to illustrate the principles and rules discussed in the text.

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PRACTICAL ARITHMETIC.

DEFINITIONS.

1. Arithmetic is that branch of mathematics which treats of quantity and its relations, when expressed by number.

2. Mathematics is the science of quantity, and embraces arithmetic, algebra, geometry, etc.

3. Quantity is anything which may be measured; as, distance, weight, time.

4. A number is either a single unit, or a collection of units; as, 1, 2, 5, 95; 4 men, 7 dollars, 9 houses.

5. A unit is a single thing; as, 1, 1 man, 1 book.

6. A number is abstract or concrete.

7. An abstract number is a collection of abstract units; as, 4, 7.

8. An abstract unit is simply *one*; as, 1.

9. A concrete unit denotes the kind of thing expressed; as, 1 man, 3 cents, 5 apples.

FUNDAMENTAL RULES.

10. The fundamental rules of arithmetic are *Notation* and *Numeration*, *Addition*, *Subtraction*, *Multiplication*, *Division*.

11. All the operations of arithmetic are based upon one or more of these fundamental rules.

NOTATION AND NUMERATION.

12. **Notation** is the art of writing numbers.

13. In arithmetic numbers are expressed by means of certain characters. These characters are either figures or letters.

14. Two systems of notation are sometimes used : the *Arabic*, and the *Roman*.

15. In the Arabic system numbers are expressed by means of the following characters, called digits :

one.	two.	three.	four.	five.	six.	seven.	eight.	nine.	naught.
1	2	3	4	5	6	7	8	9	0

16. The first nine digits are **significant figures**, because they express value when standing alone.

17. Naught expresses no value when standing alone ; but when used in connection with significant figures it performs an important office in the Arabic system.

18. The Arabic system of notation is based on a scale of TEN ; that is, *ten* units of one order make *one* unit of the next higher order throughout this system.

19. The *orders* increase in value from units' place toward the left, each order consisting of *one* figure.

Illustration. — In the number 729,

The figure 9 belongs to the order of *units*;

The figure 2 belongs to the order of *tens*;

The figure 7 belongs to the order of hundreds.

20. A period consists of *three* orders, except the left-

hand period, which often contains only two orders, or even *one*.

21. Each significant figure may have a *simple* value, or a *local* value.

22. The **simple value** of a figure is the value it expresses when standing alone, or in units' place ; as, 7, 9, 4.

23. The **local value** of a figure is the value it expresses when used in connection with other figures to express a number.

Illustration.—In the number 423,

The figure 3 expresses simple value ;

The figures 2 and 4 express local values.

24. The several orders and periods are shown below :

7	Hundreds of Sextillions.	1	Hundreds of Quadrillions.	2	Hundreds of Billions.	3	Hundreds of Millions.	4	Hundreds of Thousands.	7	Hundreds.
4	Tens of Sextillions.	7	Tens of Quadrillions.	1	Tens of Billions.	5	Tens of Millions.	4	Tens of Thousands.	8	Tens.
2	Sextillions.	5	Quadrillions.	3	Billions.	6	Millions.	5	Thousands.	9	Units.
{			{			{			{		
8th Period.			6th Period.			4th Period.			2d Period.		
Sextillions.			Quadrillions.			Billions.			Thousands.		
{			{			{			{		
7th Period.			5th Period.			3d Period.			1st Period.		
Quintillions.			Trillions.			Millions.			Units.		
{			{			{			{		
0			8			4			3		
3			0			5			4		
6			9			6			5		
{			{			{			{		
Hundreds of Quintillions.			Hundreds of Trillions.			Hundreds of Millions.			Hundreds of Thousands.		
{			{			{			{		
0			8			4			3		
3			0			5			4		
6			9			6			5		

This number is read, Seven hundred and forty-two sextillion, thirty-six quintillion, one hundred and seventy-five quadrillion, eight hundred and nine trillion, two hundred and thirteen billion, four

hundred and fifty-six million, three hundred and forty-five thousand, seven hundred and eighty-nine.

24 a. The value expressed by any number depends upon two things :

1st. Upon the number of units expressed by each digit.

2d. Upon the position of each digit with reference to units' place.

EXERCISES.

1. Write five units of the first, or units order.
2. Write eight units of the second, or tens order.
3. Write five units of the second and eight units of the first order.
4. Write six units of the third, or order of hundreds.
5. Write eight hundred, five tens, and six units.
6. Write fifty-eight ; eighty-five.
7. Write four hundred and seventy-five.
8. Write four hundred and five.
9. Write forty-five.

RULE FOR WRITING NUMBERS.

RULE. — *Write the figure expressing the number of units in the place expressing the given order. Fill vacant orders with ciphers.*

1. Four in the second order expresses how many units of the first order?

2. Write four units of the second order.
3. Five units of the third expresses how many units of the first? Of the second?
4. Write five units of the third order. Of the second order. Of the first.
5. Write five units of the first, second, and third, as one number.
6. Write seven units of the third, and two of the first, as one number. In this case, what character fills the second order? Why?
7. Name the eight periods given in the scale.
8. Name each order in each period.
9. Name the several orders consecutively, from the order of units to the order of sextillions.
10. How many orders are required to express hundred-thousands?
11. How many orders to express millions? How many periods?
12. How many orders to express tens of millions? Tens of billions?
13. How many significant figures are required to express eleven hundred million, seven thousand and twenty-three? What orders do they occupy? What orders are filled with ciphers?
14. Which orders are filled with ciphers, and which with significant figures, in writing forty-nine hundred thousand and seventy-seven?
15. Write thirty-two million, seventy-seven thousand and twenty-two.

RULE FOR READING NUMBERS.

RULE. — *Divide the number into periods, and read the periods in succession, beginning at the left, giving the name of each period except the last.*

EXERCISES.

1. How many periods in the number 723415?
2. How many periods in 12384767? How many orders?
3. How many thousands in 925682?
4. How many ten-thousands in 72841? In 74081?
5. What orders are filled with ciphers in writing :
 Twelve hundred and five?
 Sixty-one thousand and six?
 Four hundred thousand and one?
 Nine hundred and forty-nine?
 One thousand and one?

26. Numeration is the art of reading numbers when expressed by written characters.

27. Each removal of a significant figure one place further to the left multiplies the value expressed by it by *ten*.

28. Each removal of a significant figure one place further to the right divides the value expressed by it by *ten*.

29. There are two methods of reading numbers by periods.

a. In the *French Method* a period consists of *three* orders.

b. In the *English Method* a period consists of *six* orders.

In the *French Method* the scale of the period is one thousand; that is, any order of one period is one thousand times as great as the corresponding order of the next lower period. Thus: *Ten million* is *one thousand* times as great as *ten thousand*; *ten billion* is *one thousand* times as great as *ten million*.

In the *English Method* the scale of the period is one million; that is, any order above millions is one million times as great as the corresponding order of the next lower period. Thus: *Ten billion* is *one million* times as great as *ten million*; *ten trillion* is *one million* times as great as *ten billion*.

EQUIVALENTS.

ENGLISH SYSTEM.	FRENCH SYSTEM.
One million equals . . .	One million.
One billion equals . . .	One thousand billion.
One trillion equals . . .	One million trillion.
One quadrillion equals .	One billion quadrillion.
One quintillion equals .	One trillion quintillion.
One sextillion equals . .	One quadrillion sextillion.
One septillion equals . .	One quintillion septillion.
One octillion equals . .	One sextillion octillion.

ENGLISH PERIODS.	FRENCH PERIODS.	HIGHER PERIODS.
1st. Units.	1st. Units.	7th. Quintillions.
2d. Millions.	2d. Thousands.	8th. Sextillions.
3d. Billions.	3d. Millions.	9th. Septillions.
4th. Trillions.	4th. Billions.	10th. Octillions.
5th. Quadrillions.	5th. Trillions.	11th. Nonillions.
	6th. Quadrillions.	12th. Decillions.

30. In both the English and the French systems the names of the corresponding periods above quadrillions are the same. The French system of reading numbers is the one commonly used in this country.

ROMAN SYSTEM.

31. In the Roman System numbers are expressed by means of letters. These seven characters are given below :

Values:	one.	five.	ten.	fifty.	one hundred.	five hundred.	one thousand.
Letters:	I.	V.	X.	L.	C.	D.	M.

GENERAL LAWS.

I. Repeating a letter repeats its value.

	one.	two.	three.	ten.	twenty.	thirty.
Thus:	I.	II.	III.	X.	XX.	XXX.

II. The value of a letter written after one of greater value, is added to the greater value.

	six.	eleven.	fifteen.	sixteen.	sixty.
Thus:	VI.	XI.	XV.	XVI.	LX.

III. The value of a letter written before one of greater value, is taken from the greater value.

	four.	nine.	forty.	ninety.
Thus:	IV.	IX.	XL.	XC.

IV. The value of a letter written between two letters of greater value, is read in connection with the letter following it.

	nineteen.	fifty-nine.	fourteen.
Thus:	XIX.	LIX.	XIV.

V. A dash placed over a letter increases its value one thousand times. $\bar{M}$ one million.

EXERCISES.

1. Write eighteen hundred and eighty.
2. Write seventeen hundred and seventy-six.
3. Write fourteen hundred and ninety-two.
4. Write seventeen hundred and thirty-two.
5. Write eighteen hundred and twelve.
6. Write eighteen hundred and twenty.
7. Write sixteen hundred and eighty-eight.
8. Write seventeen hundred and eighty-nine.
9. Write the year of your birth.

GENERAL REVIEW.

a. DEFINE

1. Arithmetic.
2. Mathematics.
3. Quantity.
4. Number.
5. Unit.
6. Abstract number.
7. Concrete number.
8. Abstract unit.
9. Concrete unit.
10. Numeration.
11. Notation.

1. Scale.
2. Period.
3. Order.

b. NAME

12. The fundamental rules.
13. The two systems of notation.
14. Characters used in each.
15. The laws of each system.
16. The significant digits.
17. Why called significant?
18. The orders in a period.
19. The several periods.
20. The two systems of numeration.
21. The periods in each system.
22. Number of orders in each period.

C. STATE THE DIFFERENCE BETWEEN

23. The simple and the local value of a digit.
24. The French and English systems.
25. The scales of the two systems.
26. The names of the periods in the two systems.
27. The Arabic and Roman systems of notation.

ADDITION.

32. Addition is the process of finding the **sum of two or more numbers.**

The **sum** of several numbers is a single number which expresses as many units as all the numbers added.

33. Only like numbers can be added.

Like numbers are such as express the same kind of unit; as, 3, 4; \$3, \$4; 3 yds., 4 yds; 3 pecks, 4 pecks; 3 tens, 4 tens; 3 hundreds, 4 hundreds.

34. The **Sign of Addition** is +, called *plus*, and denotes that the numbers between which it is placed are to be added together.

The **sign** =, called the *sign of equality*, indicates that the several numbers which precede it, taken together, equal the number which follows it, called their **sum**.

ORAL EXERCISES.

1. Two *units* and three *units* and four *units* and one *unit* make how many *units*?

2. One *ten* plus two *tens* plus three *tens* plus four *tens* make how many *tens*?

3. Five *apples* and four *apples* and three *apples* and two *apples* make how many *apples*?

4. Seven *marbles* plus two *marbles* plus one *marble* make how many *marbles*?

5. One *hundred* and three *hundred* and five *hundred* make how many *hundred*?

6. 2 *thousands* + 3 *thousands* + 4 *thousands* make how many *thousands*?

34 a. In adding, only the *result* of each addition should be named, not the digits to be added. Thus, in adding the right column in example 1, below, we say, 3, 5, 7, 8.

35. To add when the sum in each column is less than 10.

(1.)	(2.)	(3.)		
123451	12121212	1212121212		
654322	21212121	3131313131		
211222	13131313	2010201002		
12003	32323232	1010101001		
(4.)	(5.)	(6.)	(7.)	
403020	310151	201002	101010101	
121212	101203	120120	120202020	
202021	210310	202101	201010201	
130312	123021	132332	133333233	

(8.)	(9.)	(10.)	(11.)
201201	30125	40404	1010101
121021	1200	4140	202020
120212	2110	10011	121212
202121	10121	2111	2010101
131313	21212	11210	1212121
224131	2021	21012	1010101

12. $4 + 1 + 2 + 1 + 1$ are how many?
13. $1 + 2 + 1 + 2 + 1$ are how many?
14. $2 + 2 + 2 + 2 + 1$ are how many?
15. $1 + 1 + 2 + 1 + 2$ are how many?
16. $1 + 3 + 3 + 1 + 1$ are how many?
17. $1 + 1 + 3 + 2 + 1 + 1$ are how many?

The following exercises are calculated to give facility and accuracy in adding :

1. Add by 2's from 1 to 101.
2. Add by 2's from 2 to 100.
3. Add by 2's from 3 to 103.
4. Add by 2's from 4 to 100.
5. Add by 2's from 5 to 95.
6. Add by 2's from 6 to 210.
7. Add by 2's from 7 to 99.
8. Add by 2's from 8 to 98.
9. Add by 2's from 9 to 99.
10. Add by 3's from 1 to 94.
11. Add by 3's from 2 to 100.
12. Add by 3's from 3 to 102.

13. Add by 4's from 3 to 97.
14. Add by 5's from 4 to 104.
15. Add by 6's from 5 to 101.
16. Add by 7's from 6 to 97.
17. Add by 8's from 7 to 103.
18. Add by 9's from 8 to 107.
19. Add by 9's from 10 to 109.
20. Add by 9's from 11 to 110.
21. Add by 10's from 1 to 201.
22. Add by 10's from 2 to 202.
23. Add by 10's from 3 to 203.
24. Add by 10's from 4 to 204.
25. Add by 10's from 5 to 205.
26. Add by 11's from 1 to 122.
27. Add by 11's from 3 to 124.
28. Add by 11's from 5 to 126.
29. Add by 11's from 7 to 128.
30. Add by 11's from 9 to 130.
31. Add by 12's from 1 to 145.
32. Add by 12's from 3 to 147.
33. Add by 12's from 5 to 149.
34. Add by 12's from 7 to 151.
35. Add by 12's from 9 to 153.
36. Add by 12's from 2 to 158.
37. Add by 12's from 4 to 172.
38. Add by 12's from 6 to 186.
39. Add by 12's from 8 to 200.
40. Add by 12's from 10 to 214.
41. Add by 13's from 1 to 170.

42. Add by 13's from 3 to 185.
43. Add by 13's from 5 to 200.
44. Add by 14's from 2 to 198.
45. Add by 14's from 4 to 214.
46. Add by 14's from 6 to 230.
47. Add by 15's from 1 to 226.
48. Add by 15's from 3 to 243.
49. Add by 15's from 2 to 257.
50. Add by 15's from 4 to 274.

35 a. To add when the sum in any column equals or exceeds 10.

36. Since only units of the same *order* can be added, it is necessary to write the numbers to be added so that units of the same order will stand in the same column.

37. When the sum of the units in any order exceeds *nine*, the excess is units of the next higher order, and must be added to that column.

38. Since the excess of units above *nine*, in any column, is added to the next higher order, we begin in addition with the lowest order of units, then add the tens, hundreds, and so on.

39. Proof of addition consists in adding the columns in a reverse order; that is, from the top to the bottom. As the combinations come in a different order, the same error is not likely to occur.

Suggestion.—Let the pupils embody the directions in **36**, **37**, **38**, **39**, in the form of a Rule.

MISCELLANEOUS PROBLEMS.

1. A man bought a carriage for \$217, a horse for \$175, and a harness for \$62; what did they all cost?

2. In the town A there are 2754 inhabitants; in the town B, 5781 inhabitants; in the town C, 3116 inhabitants. What is the population of the three towns?

3. A man deposited in bank on Monday \$314; on Tuesday, \$115; on Wednesday, \$72; on Thursday, \$227; on Friday, \$91; on Saturday, \$412. How much did he deposit during the week?

4. I paid \$25 for each of seven book-cases; how much did they all cost?

5. If 1250 bushels of wheat cost \$971, and 429 bushels of oats cost \$136, and 1119 bushels of rye cost \$991, how many bushels of grain were there? and how much did it all cost?

6. At the election for President in 1876, the popular vote for Tilden was 4284757; for Hayes, 4033950; for Cooper, 81740; for G. C. Smith, 9522; and scattering, 2636. What was the total vote polled?

7. The crop in the United States for the year 1877 was as follows: Wheat, 364194186 bushels; rye, 21170100 bushels; barley, 34441400 bushels; oats, 406394000 bushels; potatoes, 170092000 bushels. What was the total yield?

8. The crop for the year 1878 was estimated by the Department of Agriculture to be 425000000 bushels of wheat, 42000000 bushels of barley, 26000000 bush-

els of rye, 410000000 bushels of oats, 1746325000 bushels of corn, and 129000000 bushels of potatoes. Find the total estimate.

9. In the year 1870 the total debt of the States was \$352866698; of the counties, \$187565540; of towns and cities, \$328244520. What was the total indebtedness of the country?

10. In the years 1877-8 the following cities were indebted in the following amounts. Find their total indebtedness:

(a.)		(b.)	
Baltimore,	\$9257445	Philadelphia,	\$60781984
Charleston,	4716754	Pittsburg,	13862669
Nashville,	1712685	San Francisco,	4869000
New Orleans,	20778868	Cincinnati,	23389000
New York,	<u>129021244</u>	Cleveland,	<u>7704164</u>

(c.)		(d.)	
Washington,	\$23310146	Buffalo, N. Y.,	\$7509257
St. Louis,	22739905	Hartford, Ct.,	2152000
Providence, R.I.	10158726	Boston,	26159777
Jersey City,	14217435	Toledo,	3778849
Detroit,	<u>1207400</u>	Memphis,	<u>4906974</u>

11. The coinage of the United States Mint in the year 1879 was as follows: Gold coins, \$40986912; silver coins, \$27227882; and minor coins, \$97798. What was the value of the total coinage for that year?

12. In the following example let the pupil add columns A, B, C, as separate examples; then add sec-

tions 1, 2, 3, as separate examples ; finally, add columns A, B, and C, as *one* example :

Find the sum in each of the 34 examples given below :

A.		B.	C.
123456		789234	567891234
234567		891345	678912345
345678		912468	532179041
456789		123246	813574567
(1.) 577891	(4.) 234035	(7.) 792541013	
678912	345502	478107954	
789123	456100	301220710	
891234	567234	567890123	
912345	678002	345678912	
978643		152345	789015807
251346		879456	497605432
512364		798567	274031463
134687		925678	801375010
(2.) 346879	(5.) 251789	(8.) 570132934	
468791	523890	432045601	
687915	234902	341347902	
879152	643013	165453379	
234567	891321	613226795	
121212		123132	767676767
232323		232312	656565656
(3.) 343434	(6.) 343213	(9.) 545454545	
454545	454231	232323232	
565656	565133	141414141	
676767	676321	373737373	

SUBTRACTION.

40. Subtraction is the process of finding the difference between two numbers.

41. The number to be subtracted is called the **Subtrahend**.

42. The number from which the subtrahend is to be taken is called the **Minuend**.

43. The number found by subtracting the subtrahend from the minuend is called the **Difference**, or **Remainder**.

44. The **Sign of Subtraction** is —, and is read **minus**. The minus sign placed between two numbers indicates that the first number is to be diminished by the second.

45. The **Minuend** equals the sum of the remainder and subtrahend.

46. Hence, to prove subtraction add the remainder and subtrahend; if the sum is the minuend, the work is correct.

47. Units of one order cannot be taken from units of a different order.

48. Hence, in Subtraction, units of the same order must stand in the same column.

49. To find the difference between two numbers when each digit in the minuend is greater than its corresponding digit in the subtrahend.

1. Find the difference between 9 units and 7 units.
2. Eight units minus four units are how many?

3. Twelve units minus 5 units are how many?
4. Seven apples minus 3 apples are how many?
5. Five less 2 are how many?
6. What is the difference between 9 bushels and 5
• bushels?
7. Find the difference between 8 acres and 3 acres.
8. Can four horses be subtracted from 7 acres?

Why?

9. Can 4 horses be subtracted from 7 horses?

Why?

10. Can 4 acres be taken from 7 acres? Why?
11. Find the difference between 6 units and 3 units of the same denomination, and verify the result.
12. Write a rule for performing subtraction when each digit in the minuend exceeds its corresponding digit in the subtrahend.

$$\begin{array}{r} 13. \text{ From } 27 \\ \text{Take } 16 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 14. \text{ From } 72 \\ \text{Take } 61 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 15. \text{ From } 67 \\ \text{Take } 21 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 16. \text{ From } 331 \\ \text{Take } 120 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 17. \text{ From } 543 \\ \text{Take } 321 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 18. \text{ From } 786 \\ \text{Take } 542 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 19. \text{ From } 987 \\ \text{Take } 432 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 20. \text{ From } 876 \\ \text{Take } 321 \\ \hline \text{Diff.} \end{array}$$

$$\begin{array}{r} 21. \text{ From } 754 \\ \text{Take } 210 \\ \hline \text{Diff.} \end{array}$$

22. From 789	23. From 678	24. From 567
Take 654	Take 456	Take 213

Diff.

Diff.

Diff.

25. From 654	26. From 765	27. From 876
Take 342	Take 354	Take 654

Rem.

Rem.

Rem.

50. To find the difference between any two numbers.

Example 1. — From 7427 subtract 5869.

Explanation. — Since 9 units cannot be taken from 7 units, 1 *ten* in the minuend must be changed to *units*. This gives 17 *units*. Subtracting 9 *units* from 17 *units* there remain 8 *units*. Since one of the two tens in the minuend has been changed to units, only 1 ten remains. From 1 *ten* we cannot subtract 6 *tens*; but by changing 1 of the 4 *hundreds* to *tens*, we shall here get 11 *tens*. Subtracting 6 *tens* the remainder is 5 *tens*. In the minuend there remain only 3 *hundreds*. From this we cannot take the 8 *hundreds* in the subtrahend; but by changing one of the 7 *thousands* to *hundreds* we can take the 8 *hundreds* in the subtrahend from 13 *hundreds* in the minuend, which leaves 5 *hundreds*. Finally, 6 thousand minus 5 thousand leaves 1 thousand. The difference, then, between 7427 and 5869 is 1558.

OPERATION.

Minuend,	7427
Subtrahend,	5869
Remainder,	1558

Proof. — Since by taking 5869 from 7427 we get 1558, if the work has been correctly performed 5869 added to 1558 must give the minuend 7427. We find that the sum of 5869 and 1558 is 7427; and hence, we infer the work is correct.

51. From this explanation we derive the following rule for subtraction :

RULE. — *Write the subtrahend beneath the minuend, so that units of the same order shall stand in the same column. Subtract each figure in the subtrahend from its corresponding figure in the minuend, and write the result in the same order below. If any figure in the minuend is less than its corresponding figure in the subtrahend, change one unit of the next higher order in the minuend to this denomination, add the units of this order, and then subtract. Proceed in a similar manner with the several orders in succession, beginning with the order of units.*

(2.)	(3.)	(4.)	(5.)
From 4321	From 5432	From 6543	From 7654
Take 1234	Take 2345	Take 3456	Take 4567
Rem. <u>3087</u>	Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>

(6.)	(7.)	(8.)	(9.)
From 4231	From 5342	From 6453	From 7564
Take 1324	Take 2435	Take 3456	Take 4657
Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>

(10.)	(11.)	(12.)
From 97864	From 86431	From 87654321
Take 69783	Take 53297	Take 12345678
Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>

(13.)	(14.)	(15.)
From 654321	From 479351	From 765642
Take 567890	Take 375986	Take 567893
Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>

(16.)	(17.)	(18.)
From 257689	From 7030492	From 3785216
Take 178791	Take 4360751	Take 2139437
Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>
(19.)	(20.)	(21.)
From 98989898	From 75757575	From 86858483
Take 69696969	Take 57575757	Take 39383736
Rem. <u> </u>	Rem. <u> </u>	Rem. <u> </u>

Perform the operations indicated in the following examples :

1. $1234 + 4321 - 3124 + 2143 - 4123$.
2. $2134 - 1324 + 3214 - 2341 + 3412 - 4123$.
3. $3124 - 1243 + 4312 - 3124 + 2143 + 1432 - 1423$.
4. $4321 - 1234 + 2143 - 1324 + 3142 - 1432 + 4132$.
5. $5467 - 4567 + 4657 - 4576 + 5764 + 6745 - 7654$.
6. $6879 + 8697 - 8796 + 7869 - 9678 + 9786 + 7698$.
7. $198 + 89 - 101 + 222 - 333 + 404 + 555 - 666 + 707$.
8. $987 - 789 + 879 - 978 + 798 + 897 + 123 - 321$.
9. $842 + 248 + 428 - 284 - 124 + 314 - 612 + 111$.
10. $987 + 897 + 798 - 978 - 879 - 789$.

11. Find the sum of any two numbers, and their difference.

12. From the sum of any two numbers take their difference.

13. To the difference of any two numbers add their sum.

14. To half the sum of any two numbers add half their difference.

15. From half the sum of any two numbers take half their difference.

16. To the sum of two numbers add the sum of any other two numbers.

17. Find the difference between the sum of two numbers and the sum of two other numbers.

18. Find half the sum of any two numbers.

19. To half the first add half the second of the *same* two numbers.

20. From half the first take half the second of the *same* two numbers.

21. Work six examples similar to the *nineteenth*.

22. Work six examples similar to the *twentieth*.

23. From the greater of any two numbers take their difference.

24. To the less of any two numbers add their difference.

25. The less of any two numbers plus their difference equals what?

26. The greater of two numbers minus their difference equals what?

27. The greater of two numbers minus the less equals what?

28. The greater of two numbers is 7894, and the less is 4326. Find their difference.

29. The greater of two numbers is 8749, and the less is 3462. Find their difference.

30. The greater of two numbers is 7984, and the less is 2634. Find their difference.

31. The greater of two numbers is 7984, and their difference is 4327. Find the less number.

32. The greater of two numbers is 4994, and their difference is 1991. Find the less number.

33. The greater of two numbers is 6116, and their difference is 1661. Find the less number.

34. The less of two numbers is 1827, and their difference is 941. Find the greater number.

35. The less of two numbers is 2442, and their difference is 4224. Find the greater number.

36. Find the greater of two numbers, one of which is 1570, and whose difference is 1750.

37. What number must be added to 2116 to make 6121?

38. What number must be added to 1621 to make 2611?

39. What number must be added to 2611 to make 6211?

40. What number must be added to 1216 to make 1612?

41. What number taken from 8142 leaves 4281?

42. What number taken from 4218 leaves 2418?

43. What number taken from 4821 leaves 2148?

44. One of two numbers is 7337, and their difference 3773. Find the other.

45. One of two numbers is 1001, and their difference, 109. Find the other.

46. One of two numbers is 2002, and their difference, 1221. Find the other.

47. One-half the sum of two numbers is 1818, and one-half their difference, 1188. Find their sum and their difference.

48. One-half the sum of any two numbers increased by one-half their difference equals what?

49. Write two numbers, neither of which exceeds two thousand, and find their difference.

50. Write two numbers, neither of which exceeds ten thousand, and find both their sum and their difference.

51. How many times can you subtract 1214 from 6070?

52. The First Book of the *Æneis* contains 1065 lines, and the Second Book contains 1094 lines. How many more lines does the one contain than the other?

53. In the year 1862 the population of Boston was 177812; that of New York was 805657. How much did the population of New York exceed that of Boston?

54. The number of square miles in the State of Missouri is given as 65000, the number of square miles in the State of New York is given as 47000. How much larger is the State of Missouri than the State of New York?

55. Light moves at the rate of 185000 *miles* in one

second of time, and sound at the rate of 1142 *feet* in one second. Can you tell how much farther light moves in one second than sound? Why?

56. From the sum of 18724 and 17842 take the sum of 12784 and 12847.

57. The circumference of the Earth is given as 24956 miles, and its diameter as 7912 miles. How many miles does the circumference exceed the diameter?

58. Find the sum of the consecutive numbers from 5 to 20, and also of the consecutive numbers from 1 to 18; and find the difference between these sums.

59. Subtract by 2's from 101 to 1.

Thus: 101 less 2 is 99; 99 less 2 is 97; 97 less 2 is 95.

60. Subtract by 2's from 100 to 2.

61. Subtract by 2's from 103 to 3.

62. Subtract by 2's from 100 to 4.

63. Subtract by 2's from 95 to 5.

64. Subtract by 2's from 210 to 6.

65. Subtract by 2's from 99 to 7.

66. Subtract by 2's from 98 to 8.

67. Subtract by 2's from 99 to 9.

68. Subtract by 3's from 94 to 1.

69. Subtract by 3's from 100 to 2.

70. Subtract by 3's from 102 to 3.

71. Subtract by 4's from 97 to 2.

72. Subtract by 5's from 104 to 4.

73. Subtract by 6's from 101 to 5.

74. Subtract by 7's from 97 to 6.

75. Subtract by 8's from 103 to 7.

76. Subtract by 9's from 107 to 8.
77. Subtract by 9's from 109 to 10.
78. Subtract by 9's from 110 to 11.
79. Subtract by 10's from 201 to 1.
80. Subtract by 10's from 202 to 2.
81. Subtract by 10's from 203 to 3.
82. Subtract by 10's from 204 to 4.
83. Subtract by 10's from 205 to 5.
84. Subtract by 11's from 122 to 1.
85. Subtract by 11's from 124 to 3.
86. Subtract by 11's from 126 to 5.
87. Subtract by 11's from 128 to 8.
88. Subtract by 11's from 130 to 9.
89. Subtract by 12's from 145 to 1.
90. Subtract by 12's from 147 to 3.
91. Subtract by 12's from 149 to 5.
92. Subtract by 12's from 151 to 7.
93. Subtract by 12's from 153 to 9.
94. Subtract by 12's from 158 to 2.
95. Subtract by 12's from 172 to 4.
96. Subtract by 12's from 186 to 6.
97. Subtract by 12's from 200 to 8.
98. Subtract by 12's from 214 to 10.
99. Subtract by 13's from 170 to 1.
100. Subtract by 13's from 185 to 3.
101. Subtract by 13's from 200 to 5.
102. Subtract by 14's from 198 to 2.
103. Subtract by 14's from 214 to 4.
104. Subtract by 14's from 230 to 6.
105. Subtract by 15's from 226 to 1.
106. Subtract by 15's from 243 to 3.

107. Subtract by 15's from 257 to 2.
108. Subtract by 15's from 274 to 4.
109. Subtract by 16's from 256 to 0.
110. Subtract by 16's from 300 to 12.

MULTIPLICATION.

52. Multiplication in arithmetic consists in **repeating any number a given number of times.**

53. The number to be repeated a given number of times is called the **Multiplicand.**

54. The number which shows how many times the multiplicand is to be repeated is called the **Multiplier.**

55. The number obtained by multiplying the multiplicand by the multiplier is called the **Product.**

56. The **Multiplier** and **Multiplicand** are called **Factors.**

57. The **Sign of Multiplication** is an oblique cross, thus: $\times$, and is read, **multiplied by.**

58. The **Sign of Equality**, which consists of two horizontal parallel lines, thus: $=$, is often placed between two quantities to indicate that they are equal:

$$\text{As, } 2 \times 6 = 12.$$

59. To multiply when the multiplier consists of only one figure.

Example 1. — Multiply 232 by 3.

Explanation. — 3 times 2 units gives 6 units, which we write in the order of units. 3 times 3 tens gives 9 tens, which we write in the order of tens. 3 times 2 hundreds gives 6 hundreds, which we write in the order of hundreds.	OPERATION. <table style="margin-left: auto; margin-right: 0;"> <tr> <td style="text-align: right;">232</td> <td style="text-align: left;">Multiplicand.</td> </tr> <tr> <td style="text-align: right;">3</td> <td style="text-align: left;">Multiplier.</td> </tr> <tr> <td style="text-align: right;">—</td> <td style="text-align: left;">Product.</td> </tr> <tr> <td style="text-align: right;">696</td> <td></td> </tr> </table>	232	Multiplicand.	3	Multiplier.	—	Product.	696	
232	Multiplicand.								
3	Multiplier.								
—	Product.								
696									

From this explanation we derive the following rule :

RULE. — *Multiply each figure in the multiplicand by the multiplier, beginning with the order of units, and write the result beneath, observing to place each figure in the product in its proper order.*

MULTIPLY	MULTIPLY	MULTIPLY
2. 123 by 2.	7. 321 by 2.	12. 222 by 4.
3. 132 by 2.	8. 121 by 3.	13. 111 by 4.
4. 213 by 2.	9. 112 by 3.	14. 121 by 4.
5. 231 by 2.	10. 211 by 3.	15. 211 by 4.
6. 312 by 2.	11. 223 by 3.	16. 112 by 4.

59 a. To multiply when the multiplier consists of two or more figures.

Example 1. — Multiply 323 by 32.

Explanation.—The product	OPERATION.
of 323 units by 2 units is 646 units.	323 Multiplicand.
This is the first partial product.	32 Multiplier.
The product of 323 units by 3 tens	—
is 969 tens, or 9690 units. This is	646 First partial product.
the second partial product. Finally	969 Second partial product.
646 units plus 9690 units make 10336	—
units. This is the total product of	10336 Total product.
323 by 32.	

NOTE. — Let the pupils derive a rule from this explanation, and write it out carefully, with a view to clearness and brevity.

MULTIPLY	MULTIPLY	MULTIPLY
2. 111 by 22.	6. 221 by 23.	10. 123 by 31.
3. 222 by 11.	7. 122 by 32.	11. 342 by 22.
4. 121 by 12.	8. 331 by 23.	12. 432 by 12.
5. 112 by 21.	9. 133 by 32.	13. 243 by 32.

MULTIPLY	MULTIPLY	MULTIPLY
14. 444 by 22.	18. 111 by 111.	22. 333 by 111.
15. 444 by 12.	19. 222 by 222.	23. 333 by 222.
16. 414 by 21.	20. 333 by 333.	24. 444 by 111.
17. 144 by 11.	21. 222 by 111.	25. 444 by 222.

60. To multiply an integer by an integer, generally.

Example 1. — Multiply 6543 by 3456.

OPERATION.

6543 Multiplicand.
3456 Multiplier.

Explanation.

The product of 6543 by 6 units is.....	39258	Units.
The product of 6543 by 5 tens is.....	32715	Tens.
The product of 6543 by 4 hundreds is.....	26172	Hundreds.
The product of 6543 by 3 thousands is.....	19629	Thousands.
The product of 6543 by 3456 is.....	22612608	

From this explanation we derive the following

GENERAL RULE.

Multiply the multiplicand by each figure in the multiplier, and write the first figure in each partial product directly under that figure in the multiplier which produced it. The sum of the several partial products is the product required.

NOTE. — When there is a cipher in the multiplier there will be no partial product of that order.

MULTIPLY	MULTIPLY
2. 234 by 243.	5. 345 by 354.
3. 342 by 234.	6. 435 by 453.
4. 423 by 324.	7. 453 by 354.

MULTIPLY

8. 654 by 456.
9. 564 by 465.
10. 456 by 546.
11. 5432 by 1234.
12. 6543 by 2345.
13. 7654 by 3456.

MULTIPLY

14. 8765 by 4567.
15. 9876 by 5678.
16. 8967 by 6587.
17. 6789 by 8765.
18. 9878 by 7989.
19. 8978 by 7889.

20. Find the continued product of 1, 2, 3, 4, 5, 6, 7, 8, 9.

21. Find the continued product of 1, 3, 5, 7, 9, 11, 13, 15.

22. Find the continued product of 2, 4, 6, 8, 10, 12, 14.

23. Find the continued product of 2, 4, 8, 16, 32, 64.

24. Find the continued product of 3, 7, 11, 15, 19, 21.

25. Find the product of 2 taken twice as a factor.

26. Find the product of 3 taken twice as a factor.

27. Find the product of 4 taken twice as a factor.

28. Find the product of 5 taken twice as a factor.

29. Find the product of 2 taken three times as a factor.

30. Find the product of 3 taken three times as a factor.

31. Find the product of 4 taken three times as a factor.

32. Find the product of 5 taken three times as a factor.

33. Find the product of 2 taken four times as a factor.

34. Find the product of 3 taken four times as a factor.

35. Find the product of 4 taken four times as a factor.

36. Find the product of 5 taken four times as a factor.

37. Take 6 five times as a factor.

38. Take 7 five times as a factor.

39. Take 8 six times as a factor.

40. Take 9 seven times as a factor.

41. Take 10 ten times as a factor.

42. Take 11 five times as a factor.

43. Take 12 six times as a factor.

44. Take 13 five times as a factor.

45. Take 14 four times as a factor.

46. Take 15 as a factor eight times.

47. Take 16 as a factor eight times.

48. Multiply 15 times 15 by 16 times 16.

49. Multiply 18 times 18 by 19 times 19.

50. Multiply 21 times 21 by 31 times 31.

51. At \$227 an acre, how much will 1834 acres cost?

52. At \$347 a ton, how much will 2743 tons of iron cost?

53. If you earn \$217 in one year, how much can you earn in 37 years?

54. If a man can earn \$722 in one year, how many dollars can 713 men earn in 17 years?

55. Find the cost of 1981 barrels of flour, at \$9 a barrel.

56. Find the cost of 2147 barrels of pork, at \$22 a barrel.

57. Find the cost of 7358 kegs of tobacco, at \$32 a keg.

58. Find the cost of 92143 firkins of butter, at \$17 a firkin.

59. Find the cost of 8756 tons of hay, at \$11 a ton.

60. A man bought 342 oxen, and 542 times as many sheep. How many animals did he buy?

61. When the multiplier is a **composite number** the multiplication can be performed by using the **factors of the multiplier** in succession.

Example 1. — Multiply 847 by 45.

Suggestion.—The factors of 45 are 3, 3, and 5.

	OPERATION.
	847
	5
	—
Five times 847 are.....	4235
	3
	—
Three times 5 times, or 15 times 847, are.....	12705
	3
	—
Three times 3 times 5 times, or 45 times 847, are.....	38115

In a similar manner find the product

- | | | |
|---------------------|---|-------------------------|
| 2. Of 796 by 27. | { | Factors 3, 3, and 3. |
| 3. Of 943 by 84. | | Factors 7, 3, 2, and 2. |
| 4. Of 7437 by 105. | | |
| 5. Of 89743 by 132. | | |

62. To multiply by 10, 100, 1000, etc.

Suggestion. — Annex to the multiplicand as many ciphers as there are in the multiplier.

MULTIPLY

1. 246 by 10.
2. 462 by 100.
3. 624 by 1000.

MULTIPLY

4. 7483 by 10000.
5. 4783 by 100000.
6. 8437 by 1000000.

62 a. To multiply when there are ciphers annexed to both factors.

Suggestion.—Perform the multiplication *without* the ciphers annexed, and to the product thus found annex as many ciphers as there are at the right of *both* factors.

Example 1.—Multiply 12400 by 3400.

Explanation.—The product of 124 by 34 is 4216. Annexing *four* ciphers, the product is 42160000.

MULTIPLY

2. 340 by 250.
3. 5370 by 530.
4. 6540 by 560.
5. 7650 by 650.
6. 6700 by 1200.
7. 7900 by 6900.
8. 9800 by 8900.

MULTIPLY

9. 23400 by 3400.
10. 345000 by 4500.
11. 456700 by 56000.
12. 567800 by 670000.
13. 678000 by 567000.
14. 789000 by 6780000.
15. 8950000 by 9800000.

63. The multiplier is always an abstract number, and the product is always of the same denomination as the multiplicand.

64. Multiplication may be proved by dividing the product by either of the factors which produced it; the quotient, if the work is correct, must be the other factor.

NOTE.—If thought necessary the teacher can omit the *proof* in the first 49 examples, until the pupils shall have mastered Art. 79.

Find the product of

1. 101 by 101, and prove the work.
2. 202 by 101, and prove the work.
3. 202 by 202, and prove the work.
4. 110 by 101, and prove the work.
5. 220 by 202, and prove the work.
6. 220 by 110, and prove the work.
7. 220 by 220, and prove the work.
8. 303 by 101, and prove the work.
9. 303 by 202, and prove the work.
10. 303 by 303, and prove the work.
11. 330 by 110, and prove the work.
12. 330 by 220, and prove the work.
13. 330 by 330, and prove the work.
14. 330 by 303, and prove the work.
15. 330 by 202, and prove the work.
16. 330 by 101, and prove the work.
17. 111 by 111, and prove the work.
18. 222 by 111, and prove the work.
19. 222 by 222, and prove the work.
20. 333 by 111, and prove the work.
21. 333 by 222, and prove the work.
22. 504 by 405, and prove the work.
23. 540 by 450, and prove the work.
24. 504 by 450, and prove the work.
25. 550 by 540, and prove the work.
26. 550 by 504, and prove the work.
27. 505 by 405, and prove the work.
28. 505 by 504, and prove the work.
29. 555 by 444, and prove the work.

30. 666 by 555, and prove the work.
31. 555 by 220, and prove the work.
32. 555 by 111, and prove the work.
33. 555 by 222, and prove the work.
34. 666 by 111, and prove the work.
35. 666 by 222, and prove the work.
36. 666 by 333, and prove the work.
37. 666 by 444, and prove the work.
38. 555 by 333, and prove the work.
39. 535 by 353, and prove the work.
40. 533 by 355, and prove the work.
41. 646 by 464, and prove the work.
42. 545 by 454, and prove the work.
43. 636 by 363, and prove the work.
44. 656 by 565, and prove the work.
45. 534 by 435, and prove the work.
46. 787 by 878, and prove the work.
47. 888 by 777, and prove the work.
48. 877 by 788, and prove the work.
49. 999 by 888, and prove the work.

Find the product

50. Of the factors 7, 10 and 21.
51. Of the factors 8, 11 and 22.
52. Of the factors 9, 12 and 23.
53. Of the factors 10, 13 and 24.
54. Of the factors 11, 14 and 25.
55. Of the factors 12, 15, 25 and 30.
56. Of the factors 13, 16, 26 and 40.
57. Of the factors 14, 17, 27 and 50.
58. Of the factors 15, 18, 28 and 60.

59. Of the factors 16, 19, 29 and 70.
60. Of the factors 17, 10, 22, 30 and 45.
61. Of the factors 18, 20, 33, 40 and 55.
62. Of the factors 19, 30, 44, 50 and 66.
63. Of the factors 20, 40, 55, 60 and 77.
64. Of the factors 21, 50, 66, 70 and 88.
65. Of the factors 22, 60, 77, 80 and 99.
66. Of the factors 23, 70, 88, 90 and 111.
67. Of the factors 24, 80, 99, 100 and 222.
68. Of the factors 5, 10, 15, 20, 25 and 30.
69. Of the factors 10, 14, 24, 34, 44 and 200.
70. Of the factors 20, 24, 34, 44, 54 and 300.
71. Of the factors 30, 35, 45, 55 and 400.
72. Of the factors 40, 45, 65, 70 and 500.
73. Of the factors 101, 202, 303 and 404.
74. Of the factors 202, 404 and 505.
75. Of the factors 303, 606 and 707.
76. Of the factors 404, 606 and 707.
77. Of the factors 707, 808 and 909.
78. At \$45 an acre, what will 7432 acres of land cost?
79. At \$124 apiece, what will 1723 horses cost?
80. If one barrel of flour weighs 196 pounds, find the weight of 4116 bbls.
81. Find the cost of 728 city lots, at \$79 per front foot.
82. Find the cost of 837 city lots, at \$103 per front foot.
83. What will 23 steamships cost at \$93741 apiece?
84. What will 181 schooners cost at \$12347 apiece?
85. At \$111 a head, what will 4742 mules cost?

86. At \$23 a head, what will 3114 cattle cost?
87. At \$117 per acre, what will 747 acres of land cost?
88. If one acre yields 83 bushels of corn, what will 936 acres yield?
89. If one acre yields 27 bushels of wheat, what will 1115 acres yield?
90. If one acre yields 32 bushels of oats, what will 349 acres yield?
91. Find the cost of 9116 cattle, at \$96 a head.
92. Find the cost of 1961 cattle, at \$69 a head.
93. Find the cost of 7256 miles of railroad, at \$19251 per mile.
94. Find the cost of 18322 miles of railroad, at \$27016 per mile.
95. Find the cost of 1417 cars, at \$17122 apiece.
96. Find the cost of 748 sleepers, at \$21419 apiece.
97. At 239 miles per day, how far will a train run in 2169 days?
98. At 314 miles per day, how far will a train run in 1618 days?
99. At \$289 a ton, how much will 893 tons of railroad iron cost?
100. At \$432 a ton, what will 915 tons of steel rails cost?

DIVISION.

65. Division in arithmetic is finding how many times one number is contained in another.

66. The Dividend is the number to be divided.

67. The **Divisor** is the number by which we divide.

68. The **Quotient** shows the number of times the **Divisor** is contained in the **Dividend**.

69. A **Divisor** of a number is a **Factor** of that number.

70. An **Exact Divisor** of a number is a divisor which is contained in the given number an integral number of times.

71. The **Sign of Division** ($\div$) is read **divided by**.

72. The **Remainder in division** is that part of the dividend, after division, which will not contain the **divisor**, being always less than the divisor.

73. To divide when the divisor does not exceed 9, and is an exact divisor of each figure in the dividend.

Example 1. — Divide 369 by 3.

Explanation. — Dividing any number by 3, is simply finding one-third of that number.

One-third of 3 hundreds is 1 hundred.

One-third of 6 tens is 2 tens.

One-third of 9 units is 3 units.

Hence, the quotient is 123.

OPERATION.

Divisor. Dividend.

$$\begin{array}{r} 3 \overline{)369} \\ \underline{3} \\ 6 \\ \underline{6} \\ 9 \\ \underline{9} \\ 0 \end{array}$$

123 Quotient.

74. As in multiplication the **product** is of the same denomination as the multiplicand, so in division the **quotient** is of the same denomination as the dividend.

RULE. — *Divide each figure in the dividend by the divisor, and write the quotient figure immediately beneath the figure in the dividend which produced it. The result will be the quotient required.*

In a similar manner explain each of the following examples :

- | | |
|---------------------|----------------------|
| 2. Divide 248 by 2. | 7. Divide 555 by 5. |
| 3. Divide 428 by 2. | 8. Divide 666 by 6. |
| 4. Divide 824 by 2. | 9. Divide 777 by 7. |
| 5. Divide 996 by 3. | 10. Divide 888 by 8. |
| 6. Divide 844 by 4. | 11. Divide 999 by 9. |

75. To divide when the divisor exceeds 9, but is not greater than 12.

Example 1.—Divide 83091612 by 12.

OPERATION.

Explanation.— Since the divisor is not contained in 8 ten-millions we reduce the 8 *ten-millions* to *millions* and add the 3 millions. This gives us the first partial dividend, 83 millions.

Divisor. Dividend.

12)83091612

6924301 Quotient.

One-twelfth of 83 millions is 6 millions, with a remainder of 11 millions, which we reduce to hundred-thousands.

One-twelfth of 110 hundred-thousands is 9 hundred-thousands, with a remainder of 2 hundred-thousands, which we reduce to ten-thousands, and add the 9 ten-thousands.

One-twelfth of 29 ten-thousands is 2 ten-thousands, with a remainder of 5 ten-thousands, which we reduce to thousands, and add the 1 thousand in the dividend.

One-twelfth of 51 thousand is 4 thousand, with a remainder of 3 thousand, which we reduce to hundreds, and add the 6 hundred in the dividend.

One-twelfth of 36 hundred is 3 hundred, with no remainder.

Since 12 is not contained in 1 ten, we write a cipher in the quotient to show there are no *tens*, and reduce the 1 ten to units, and add the 2 units in the dividend.

One-twelfth of 12 units is 1 unit.

Hence, 83091612 divided by 12 gives 6924301 as a quotient.

RULE.— *If the figure in the highest order in the*

dividend is not divisible by the divisor, reduce it to the next lower order, add the units of that order, and then divide.

Any remainder, after dividing any order in the dividend by the divisor, must be reduced to the next lower order and added to the units of that order, and then the division may be continued as before.

Proceed in this manner until the order of units is reached and divided, when the division will be complete.

In a similar manner explain the following examples :

- | | |
|-----------------------|-------------------------|
| 2. Divide 10234 by 2. | 6. Divide 591612 by 6. |
| 3. Divide 21249 by 3. | 7. Divide 670218 by 7. |
| 4. Divide 36736 by 4. | 8. Divide 7013456 by 8. |
| 5. Divide 45905 by 5. | 9. Divide 8135278 by 9. |

76. When the divisor is not an *exact* divisor of the dividend, there will be a remainder after division. This remainder forms the numerator of a fraction, of which the divisor is the denominator ; the integral part of the quotient and this fraction forming the complete quotient.

77. In multiplication the factors are given to find the product. In division the product, called the dividend, and one of the factors, called the divisor, are given to find the other factor, called the quotient.

78. Division is the reverse of Multiplication. Hence, to prove division, multiply the divisor by the quotient, and to this product add the remainder, if any. The result must equal the dividend.

79. To divide one number by another, generally.

Example 1. — Divide 42312472 by 8402.

Explanation. — Since the divisor consists of four figures, and the left hand figure is greater than the left hand figure of the dividend, we must use five figures as the *first partial dividend*. Now, 8 thousand is contained in 42 millions about 5 *thousands* times; and 5 times 8402 gives 42010 to be subtracted from the dividend, leaving 302. Bringing down the next figure of the dividend, the *second partial dividend* is 3024. The divisor is not contained in this. Writing a cipher in the quotient to show there are no *hundreds*, and bringing down the next figure of the dividend (which is the same as reducing the remainder to the next lower denomination and adding the units of that denomination), the *third partial dividend* is 30247. The divisor is contained in this dividend 3 *tens* times. Multiplying the divisor by this quotient figure and subtracting the product from the partial dividend, and bringing down the next figure of the dividend, the *fourth partial dividend* is 50412. The divisor is contained in this partial dividend 6 *units* times. Multiplying the divisor by this quotient figure, and subtracting, there is no remainder. Hence, $42312472 \div 8402 = 5036$.

OPERATION.

Di- visor.	Divi- dend.	Quo- tient.
8402)	42312472	(5036
	42010	

30247

25206

50412

50412

0 Remainder.

Proof. — If 5036 expresses the number of times 8402 is contained in 42312472, then 8402 multiplied by 5036 must give a product equal to the dividend, 42312472. Multiplying 8402 by 5036 we get 42312472.

RULE. — I. Write the divisor at the left of the dividend, and the several figures in the quotient as they are obtained, at the right.

II. Divide the fewest number of figures in the dividend that will contain the divisor, and write the result in the quotient.

III. Multiply the divisor by this quotient figure, sub-

tract the product from those figures of the dividend used in obtaining it, and annex the next figure of the dividend to the remainder.

IV. Divide this new partial dividend by the divisor, multiply the divisor by this quotient figure, and subtract the product from this partial dividend. Annex the next figure of the dividend to this remainder, and divide as before.

V. When any new partial dividend does not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and divide as before.

VI. When there is a remainder after obtaining the last quotient figure, write it over the divisor, and annex this fraction to the quotient already found. This will be the complete quotient.

In a similar manner explain each of the following examples :

2. Divide 26781 by 1324.
3. Divide 378984 by 2462.
4. Divide 489465 by 3569.
5. Divide 5987427 by 4689.
6. Divide 608421 by 7894.
7. Divide 709531 by 8985.
8. Divide 800163 by 9876.
9. Divide 901046 by 8967.

80. Multiplication and division compared.

By comparing the operation of multiplication with the operation of division, it will be seen that the two

factors in multiplication correspond to the divisor and quotient in division ; and that the product in multiplication corresponds to the dividend in division.

In multiplication, we find that the final product is the sum of several partial products ; that each of these partial products is found by multiplying the multiplicand by some one of the figures of the multiplier.

Hence, finding the product of one number by another number consists of two distinct operations : First, finding the several partial products which result from multiplying the multiplicand by the several figures of the multiplier in succession. Second, finding the sum of these several partial products.

In the example given below, the product in Case I. becomes the dividend in Case II. ; and if the multiplicand is taken as the divisor, the multiplier becomes the quotient.

CASE I.

MULTIPLICATION.

Multiply	98765	The multiplicand.
By	56789	The multiplier.
a.	888885	Product by the units figure.
b.	790120	Product by the tens figure.
c.	691355	Product by the hundreds figure.
d.	592590	Product by the thousands figure.
e.	493825	Product by the ten-thousands figure.
	5608765585	Total product by the multiplier.

CASE II.
DIVISION.

Divide 5608765585 by 89765.

Divisor.	Dividend.	Quotient.
	98765	5608765585(56789
e.	493825	Product by the ten-thousands figure.
	670515	First new dividend.
d.	592590	Product by the thousands figure.
	779255	Second new dividend.
c.	691355	Product by the hundreds figure.
	879008	Third new dividend.
b.	790120	Product by the tens figure.
	888885	Fourth new dividend.
a.	888885	Product by the units figure.
	000000	Remainder.

NOTE.— By comparing carefully the two preceding operations the pupils will see that one of them is just the reverse of the other.

The partial product marked (a) in the first case is the same as the product marked (a) in the second case.

The partial product marked (b) in the first case is the same as the product marked (b) in the second case.

The partial product marked (c) in the first case is the same as the product marked (c) in the second case.

The partial product marked (d) in the first case is the same as the product marked (d) in the second case.

The partial product marked (e) in the first case is the same as the product marked (e) in the second case.

81. To divide by 100, 1000, 10000, 100000.

RULE.— *Remove the point at the right of units as many orders to the left of units as there are ciphers annexed to the divisor.*

Explain the application of the rule to the following examples :

1. Divide 1234 by 100.
2. Divide 2345 by 1000.
3. Divide 34567 by 10000.
4. Divide 4567890 by 100000.

82. GENERAL PRINCIPLES OF DIVISION.

First Principle.

Multiplying the dividend by any number multiplies the quotient by that number.

Illustration.—8 is contained in 24 three times; and in 3 times 24 it is contained 9 times; that is, 3 times 3 times.

Second Principle.

Multiplying the divisor by any number divides the quotient by that number.

Illustration.—8 is contained in 72 nine times; but 3 times 8, or 24, is contained in 72 only 3 times; that is $\frac{1}{3}$ of 9 times.

Third Principle.

Multiplying both divisor and dividend by the same number has no effect on the quotient.

Illustration.—8 is contained in 24 three times; and 4, which is one-half of 8, is contained in 12, which is one-half of 24, just *three* times.

Fourth Principle.

Dividing the dividend by any number divides the quotient by that number.

Illustration.—8 is contained in 72 nine times; but it is contained in 24, which is one-third of 72, only *one-third* of nine times; that is three times.

Fifth Principle.

Dividing the divisor by any number multiplies the quotient by that number.

Illustration. — 8 is contained in 24 three times; but 4, which is one-half of 8, is contained in 24 twice three, or 6 times.

Sixth Principle.

Dividing both divisor and dividend by the same number has no effect on the quotient.

Illustration. — 8 is contained in 72 nine times; and 4, which is one-half of 8, is contained in 36, which is one-half of 72, just *nine* times.

By means of these principles, explain the following examples :

1. How many *more* times is 2 contained in 12, than in 6? Why?
2. How many *more* times is 3 contained in 18, than in 6? Why?
3. How many times *less* is 4 contained in 24, than in 72? Why?
4. How many times less is 5 contained in 35, than in 95? Why?
5. How many more times is 7 contained in 28, than 21 in 84? Why?
6. How many more times is 11 contained in 55, than 33 in 99? Why?
7. How many times less is 9 contained in 63, than 18 in 162? Why?

8. Does 144 divided by 12 give a greater, or less, result than 72 divided by 6?

9. Does 338 divided by 26 give a greater, or less, result than 169 divided by 13?

10. Divide 12345 by 23, and prove the result.

11. Divide 23456 by 32, and prove the result.

12. Divide 345678 by 43, and prove the result.

13. Divide 456789 by 54, and prove the result.

14. Divide 54321 by 789, and prove the result.

15. Divide 65432 by 678, and prove the result.

16. Divide 876543 by 567, and prove the result.

17. Divide 987654 by 4567, and prove the result.

18. Divide 896745 by 5476, and prove the result.

19. Divide 785634 by 765, and prove the result.

20. Divide 1090801 by 20301, and prove the result.

21. Perform the operations $243 \times 672 \div 469$.

22. Perform the operations $324 \times 126 \div 214$.

23. Perform the operations $989 \times 898 \div 789$.

24. Perform the operations $987 \times 789 \div 879$.

25. What is the quotient of 89543 divided by 4321?

26. What is the quotient of 98452 divided by 3412?

27. What is the quotient of 1091820 divided by 12345?

28. $1875 \div 25 + 20 \times 23 = \text{what?}$

29. $864 + 64 \div 8 \times 400 = \text{what?}$

30. 1578 barrels of beef cost \$23670; find the cost of one barrel.

31. Divide 38504275 by 3; by 72; by 465.

32. Divide 1246859 by 2; by 84; by 719.

33. Divide 2354075 by 7; by 19; by 439.

34. Divide 9806257 by 4 ; by 57 ; by 171.
35. Divide 2950296 by 5 ; by 27 ; by 981.
36. Divide 276942 by 457. Ans. 606.
37. Divide 789591 by 213. Ans. 3707.

UNITED STATES CURRENCY.

83. By United States Currency is understood the money of the United States.

84. This currency is based on the Decimal System.

Ten units of one denomination make a unit of the next higher, as in whole numbers.

84 a. The coins used in this system are composed of gold, silver, copper, and nickel.

Value of the several gold coins :

The **Double Eagle** equals in value **Twenty Dollars**.

The **Eagle** equals in value **Ten Dollars**.

The **Half-Eagle** equals in value **Five Dollars**.

The **Quarter-Eagle** equals in value **Two and one-half Dollars**.

The **Three Dollar** and the **Dollar** gold coins are not much used.

Value of the several silver coins :

The **Dollar** equals in value **One hundred cents**.

The **Half Dollar** equals in value **Fifty cents**.

The **Quarter Dollar** equals in value **Twenty-five cents**.

The **Dime** equals in value **Ten cents**.

The **Half-Dime** equals in value **Five cents**.

The copper coins are the **one** and the **two** cent pieces.

The nickel coins are the **three** and the **five** cent pieces.

The **Dollar Sign** is written **\$**.

The decimal point (.) separates dollars and cents.

The **Mill**, used in calculations, is not coined.

85. To change any number of units to an equivalent value expressed in terms of any lower denomination.

1. Change 25 dimes to cents.

Equivalent. — 10 cents make one dime.

2. Change 43 dimes to cents.
3. Change 56 dimes to cents.
4. Change 27 dollars to dimes.

Equivalent. — 10 dimes make one dollar.

5. Change 32 dollars to dimes.
6. Change 45 dollars to dimes.
7. Change 57 dollars to cents.

Equivalent. — 100 cents make one dollar.

8. Change 71 dollars to cents.
9. Change 73 dollars to dimes ; to cents.
10. Change 84 dollars to dimes ; to cents.
11. Change 2 cents to mills.

Equivalent. — 10 mills make one cent.

12. Change 8 cents to mills.
13. Change 2 dollars to dimes ; to cents ; to mills.

85 a. To change any number of units of a lower denomination to an equivalent value expressed in terms of a higher denomination.

14. Change 1200 cents to dimes.

Suggestion. — There can be only *one-tenth* as many dimes as there are cents.

15. Change 40 cents to dimes.
16. Change 40 dimes to dollars.
17. Change 500 cents to dimes ; to dollars.

85 b. To add United States Currency.

1. Add \$25, \$35, \$45.

Suggestion. — Add as in whole numbers.

2. Find the sum of \$271, \$127, \$712.

3. Find the sum of \$313, \$133, \$331.

85 c. To subtract United States Currency.

4. From \$842 subtract \$248.

Suggestion. — Subtract as in whole numbers.

5. From 73 cents subtract 37 cents.

6. From \$73.37 subtract \$37.73.

7. From \$91.91 subtract \$19.19.

85 d. To multiply United States Currency.**Suggestion.** — Multiply as in whole numbers.

(8.)	(9.)	(10.)
Multiply \$125	Multiply \$250	Multiply 481
By 8	By 4	By 9
\$1000		

(11.)	(12.)	(13.)
Multiply \$41.14	Multiply \$18.18	Multiply \$1234.56
By 7	By 12	By 3

85 e. To divide United States Currency.**Suggestion.** — Divide as in whole numbers.

14. Divide \$91.15 by 5.

15. Divide \$144.24 by 4.

16. Divide \$81.27 by 9.

17. Divide \$169.26 by 13.

18. Divide \$216.56 by 8.
19. Divide \$288.48 by 12.
20. Divide \$576.96 by 24.
21. Divide \$512.32 by 16.
22. Divide \$2142.84 by 21.
23. Divide \$4284.84 by 42.
24. What does 1 man earn, if 12 men earn \$172.80?
25. If 12 yards cost \$42.60, what did 1 yard cost?
26. Find the cost of 1 ox, if 17 oxen cost \$1445.
27. If 224 acres cost \$5600, what did 1 acre cost?
28. If 864 sheep cost \$3456, what was the cost of 1 sheep?

GREATEST COMMON DIVISOR.

86. A **Divisor** of a number is a number that will exactly divide it. A **Factor** of a number is a divisor of it. A **prime factor** cannot be resolved into factors other than itself and unity.

87. A **Common Divisor** of two or more numbers is an exact divisor of each of them.

88. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

FUNDAMENTAL PRINCIPLE.

The Greatest Common Divisor of several numbers is the product of all the factors common to the several numbers.

First. If this product contains any factor not found

in either of the numbers, it will not exactly divide that number.

Second. If this product does not contain *all* the factors common to the several numbers, it will not be the *greatest* common divisor of them.

FIRST METHOD.

89. To find the Greatest Common Divisor of several numbers by factoring.

Example 1. — Find the Greatest Common Divisor of 12, 32, 44, and 56.

Solution. — The prime factors of 12 are 2, 2, and 3.

The prime factors of 32 are 2, 2, 2, 2, and 2.

The prime factors of 44 are 2, 2, and 11.

The prime factors of 56 are 2, 2, 2, and 7.

Since 3 is not a factor of 32, 44, and 56, it can form no part of the **Greatest Common Divisor**.

For a similar reason 7 and 11 can form no part of the **Greatest Common Divisor**.

Since 2, taken twice, is the only factor common to *all* the numbers, their product, 4, is the greatest Common Divisor sought.

Example 2. — Find the Greatest Common Divisor of 36, 48, 84, and 96.

Solution. — The prime factors of 36 are 2, 2, 3, and 3.

The prime factors of 48 are 2, 2, 2, 2, and 3

The prime factors of 84 are 2, 2, 3, and 7.

The prime factors of 96 are 2, 2, 2, 2, 2, and 3.

The only factors common to these several numbers are 2, 2, and 3. Hence, 12, their product, is the Greatest Common Divisor.

3. Find the Greatest Common Divisor of 24, 38, 56, and 74.

4. Find the Greatest Common Divisor of 23, 46, 69, and 115.
5. Find the Greatest Common Divisor of 21, 63, 84, and 105.
6. Find the Greatest Common Divisor of 51, 85, 119, and 153.
7. Find the Greatest Common Divisor of 39, 91, 169, and 195.
8. Find the Greatest Common Divisor of 45, 81, 108, and 189.
9. Find the Greatest Common Divisor of 144, 360, 432, and 648.
10. Find the Greatest Common Divisor of 431, 341, and 143.

SECOND METHOD.

90. This Method of finding the Greatest Common Divisor of two or more numbers depends on the following principles :

First Principle.

A Divisor of a number is a divisor of any multiple of that number.

Demonstration. — Let a be any number, and b a divisor of it. Then $2a$, $3a$, $4a$, na , represent multiples of a . It is evident that if b is a divisor of a , that is, if b is contained in a a certain number of times, it must be contained in $2a$, $3a$, $4a$; na , *twice, three times, four times, n times as many times.*

Second Principle.

A Divisor of two or more numbers is also a divisor of their sum.

Demonstration. — Let a be any number, and b a divisor of it. Then by FIRST PRINCIPLE b is a divisor of $2a$, $3a$, $4a$, na . But b , being a divisor of a , and also of $2a$, is a divisor of $3a$, their sum. It is also a divisor of a plus $3a$, or $4a$, of a plus $4a$, or $5a$. Generally, of na plus a , or, $a(n+1)$

Third Principle.

A Divisor of two numbers is also a divisor of their difference.

Demonstration. — Let a be any number, and b a divisor of it. Then, by FIRST PRINCIPLE, a is a divisor of $4a$ and $7a$. But a is also a divisor of $3a$, their difference.

Example 1. — Find the Greatest Common Divisor of 2043 and 5221.

$$\begin{array}{rcl}
 & & \text{1st. Dividend.} \\
 \text{1st Divisor } 2043 & 5221 & (2 \\
 & 4086 & \\
 & \text{---} & \text{2d. Dividend.} \\
 \text{2d Divisor } 1135 & 2043 & (1 \\
 & 1135 & \\
 & \text{---} & \text{3d Dividend.} \\
 \text{3d Divisor } 908 & 1135 & (1 \\
 & 908 & \\
 & \text{---} & \text{4th Dividend.} \\
 \text{4th Divisor } 227 & 908 & (4 \\
 & 908 & \\
 & \text{---} &
 \end{array}$$

Explanation. — A divisor of 2043 must, by Principle First, be a divisor of 4086. The divisor sought, then, is a divisor of 4086 and 5221, and must, by Principle Third, be a divisor of the difference, 1135.

Since the divisor sought is a divisor of 1135 and 2043, it must, by the Third Principle, be a divisor of 908, their difference.

Since the divisor sought is a divisor of 908 and also a divisor of 1135, it must, by Principle Third, be a divisor of their difference, which is 227.

Proof. — Since the divisor sought is a divisor of 227, it must, by the First Principle, be a divisor of any multiple of 227, that is, of 908.

The divisor sought being a common divisor of 227 and 908, it must, by the Second Principle, be a divisor of their sum, 1135.

The divisor sought being a common divisor of 908 and 1135, it must, by the Second Principle, be a divisor of their sum, 2043.

The divisor sought being a divisor of 2043, it must, by the First Principle, be a divisor of any multiple of 2043, that is, of 4086.

And being a divisor of 2043 and 4086, it must, by the Second Principle, be a divisor of their sum, 5221.

Finally. Since no number can have a divisor greater than itself; and since the divisor sought must be a divisor of 227, 2043, and 5221, it follows that the greatest common divisor of 2043 and 5221 is 227.

2. Find the Greatest Common Divisor of 2760 and 7245. Ans. 345.

3. Find the Greatest Common Divisor of 2387 and 3255. Ans. 217.

4. Find the Greatest Common Divisor of 8652 and 20909. Ans. 721.

5. Find the Greatest Common Divisor of 768, 1408, 3200.

Suggestion. — Find the Greatest Common Divisor of two of the given numbers. Then find the Greatest Common Divisor of it and the other number.

LEAST COMMON MULTIPLE.

91. A multiple of any number is a number which will **exactly contain** it. Thus, 24, 36, 48, 72, 84, and 96, are multiples of 12.

92. A Common Multiple of two or more numbers is a number which is **exactly divisible by each** of them. Thus, 144 is a common multiple of 72, 48, 24, 16, 12, 9, 8, 6, 4, 3, 2.

93. The **Least Common Multiple** of two or more numbers is the *least* number that will contain each of them an integral number of times. Thus, 60 is the least common multiple of 30, 20, 15, 12, 10, 5, 4, 3, 2.

FUNDAMENTAL PRINCIPLES.

The Least Common Multiple of several numbers is the product of all the different prime factors of those numbers, taken the greatest number of times they are found in either of the given numbers.

94. a. The **Least Common Multiple** must contain *all* the *different* prime factors found in the numbers themselves, otherwise it would not be divisible by each of them.

b. The **Common Multiple** must not contain any prime factor a *greater* number of times than it is found in any of the numbers, otherwise it will not be the **Least Common Multiple**.

FIRST METHOD.

95. To find the **Least Common Multiple** by **factoring**.

Example 1. — Find the **Least Common Multiple** of 48, 64, 72, 96.

Solution. — The prime factors of 48 are 2, 2, 2, 2, and 3.

The prime factors of 64 are 2, 2, 2, 2, 2, and 2.

The prime factors of 72 are 2, 2, 2, 3, and 3.

The prime factors of 96 are 2, 2, 2, 2, 2, and 3.

Here, the several *different* factors are 2 and 3.

Since 2 occurs *six* times as a factor in 64, it must occur *six* times as a factor in the **Least Common Multiple**.

Since 3 occurs *twice* as a factor in 72, it must occur *twice* in the

Least Common Multiple. Hence, the Least Common Multiple of 48, 64, 72, and 96 is 576.

2. Find the Least Common Multiple of 12, 14, 18, and 21. Ans. 252.

3. Find the Least Common Multiple of 12, 21, 20, and 48.

4. Find the Least Common Multiple of 11, 13, 26, and 99.

5. Find the Least Common Multiple of 13, 39, 56, and 63.

6. Find the Least Common Multiple of 8, 10, 12, 18, and 25.

7. Find the Least Common Multiple of 4, 5, 6, 7, and 8.

8. Find the Least Common Multiple of 16, 24, 36, and 72.

9. Find the Least Common Multiple of 18, 32, 42, and 56.

10. Find the Least Common Multiple of 20, 34, 54, and 65.

96. The product of several numbers which are **prime to each other**, is their least common multiple.

97. Numbers are prime to each other when no whole number is an **exact divisor** of all of them.

SECOND METHOD.

98. This method differs from the first only in the manner of resolving the given numbers into their prime factors.

Example 1. — Find the Least Common Multiple of 18, 27, 36, 44, 56, and 63.

Suggestion.—Arrange the several numbers as below, and divide by any *prime* number that is an *exact* divisor of two or more of them; write the quotients beneath, and bring down the numbers undivided. Proceed in the division until no prime number will divide two of the given numbers.

The *product* of the several divisors, quotients, and remainders, will be the least common multiple.

OPERATION.

Divisor.	2)	18	27	36	44	56	63
	2)	9	27	18	22	28	63
	3)	9	27	9	11	14	63
	3)	3	9	3	11	14	21
	7)	1	3	1	11	14	7
		1	3	1	11	2	1

Here, 2 occurs *three* times as a factor; 3 occurs *three* times; 7 occurs *once*, and 11 *once*. Hence, $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 11$, or 16632, is the least common multiple of 18, 27, 36, 44, 56, and 63.

2. Find the Least Common Multiple of 15, 25, 35, 45.

3. Find the Least Common Multiple of 14, 28, 42, 56, 64.

4. Find the Least Common Multiple of 10, 20, 30, 40, 50.

5. Find the Least Common Multiple of 16, 20, 24, 28, 32.

6. Find the Least Common Multiple of 18, 22, 32, 44, 54.

7. Find the Least Common Multiple of 22, 33, 44, 55, 66.

BUSINESS METHODS.

99. Business Calculations may often be greatly facilitated by using the following **fractional parts of One Dollar**.

One-third of one dollar equals $33\frac{1}{3}$ cents.
One-sixth of one dollar equals $16\frac{2}{3}$ cents.
One-eighth of one dollar equals $12\frac{1}{2}$ cents.
One-twelfth of one dollar equals $8\frac{1}{3}$ cents.
One-sixteenth of one dollar equals $6\frac{1}{4}$ cents.
One-half of one dollar equals 50 cents.
One-fourth of one dollar equals 25 cents.
One-fifth of one dollar equals 20 cents.
One-tenth of one dollar equals 10 cents.
One-twentieth of one dollar equals 5 cents.

APPLICATIONS.

1. Find the cost of 64 arithmetics, at $37\frac{1}{2}$ cents apiece.

Explanation. — $37\frac{1}{2}$ is *three* times $12\frac{1}{2}$.
 $12\frac{1}{2}$ cts. is $\frac{1}{2}$ of one dollar.
 Then, $37\frac{1}{2}$ cents is $\frac{3}{2}$ of one dollar.
 64 arithmetics, at \$1, cost \$64.
 64 arithmetics, at $37\frac{1}{2}$ cents, cost $\frac{3}{2}$ of \$64,
 or \$24.

2. Find the cost of 96 readers, at $87\frac{1}{2}$ cents apiece.

Explanation. — $87\frac{1}{2}$ is *seven* times $12\frac{1}{2}$.
 $12\frac{1}{2}$ cents is $\frac{1}{2}$ of one dollar.
 Then, $87\frac{1}{2}$ cents is $\frac{7}{2}$ of one dollar.
 96 readers, at \$1, cost \$96.
 96 readers, at $87\frac{1}{2}$ cents, cost $\frac{7}{2}$ of \$96, or \$84.

3. Find the cost of 105 bushels of wheat, at $66\frac{2}{3}$ cents per bushel.

Explanation. — $66\frac{2}{3}$ is *twice* $33\frac{1}{3}$.
 $33\frac{1}{3}$ cents is $\frac{1}{3}$ of one dollar.
 Then, $66\frac{2}{3}$ cents is $\frac{2}{3}$ of one dollar.
 105 bu. of wheat, at \$1, cost \$105.
 105 bu. of wheat, at $66\frac{2}{3}$ cents, cost $\frac{2}{3}$ of
 \$105, or \$70.

4. $62\frac{1}{2}$ cents is what part of one dollar?
5. $18\frac{3}{4}$ cents is what part of one dollar? Base, $6\frac{1}{2}$.
6. $31\frac{1}{4}$ cents is what part of one dollar? Base, $6\frac{1}{2}$.
7. $43\frac{3}{4}$ cents is what part of one dollar? Base, $6\frac{1}{2}$.
8. $41\frac{2}{3}$ cents is what part of one dollar? Base, $8\frac{1}{2}$.
9. $58\frac{1}{3}$ cents is what part of one dollar? Base, $8\frac{1}{2}$.
10. $83\frac{1}{3}$ cents is what part of one dollar? Base, $16\frac{2}{3}$.
11. Find the cost of 24 pocket-knives, at $12\frac{1}{2}$ cents apiece.
12. Find the cost of 4 dozen pocket-knives, at $12\frac{1}{2}$ cents apiece.
13. Find the cost of 7 gross of steel-pens, at $62\frac{1}{2}$ cents a gross.
14. Find the cost of 48 yards of calico, at $6\frac{1}{4}$ cents a yard.
15. Find the cost of 36 yards of muslin, at $16\frac{2}{3}$ cents a yard.
16. Find the cost of 64 books, at $37\frac{1}{2}$ cents apiece.
17. Find the cost of 72 books, at $8\frac{1}{3}$ cents apiece.
18. Find the cost of 144 books, at $12\frac{1}{2}$ cents apiece.
19. Find the cost of 60 books, at $16\frac{2}{3}$ cents apiece.
20. Find the cost of 120 books, at $33\frac{1}{3}$ cents apiece.
21. Find the cost of 824 yards of silk, at \$2.50 a yard.

Solution. — $2\frac{1}{2}$ is one-fourth of 10.

The cost of 824 yds. at \$10 is \$8240;

The cost of 824 yds. at $\$2\frac{1}{2}$ is $\frac{1}{4}$ of \$8240, or \$2060.

22. Find the cost of 1124 yards of silk, at \$2.50.
23. Find the cost of 1848 yards of silk, at \$2.50.
24. Find the cost of 1848 yards of cloth, at \$1.25.

Solution. — $1\frac{1}{4}$ is *one-eighth* of 10.

The cost of 1848 yards, at \$10, is \$18480.

The cost of 1848 yards, at $1\frac{1}{4}$, is $\frac{1}{8}$ of \$18480, or \$2310.

25. Find the cost of 51 pounds of nails, at $3\frac{1}{8}$ cents a pound.

Solution. — $3\frac{1}{8}$ is *one-third* of 10.

The cost of 51 pounds, at 10 cents, is \$5.10.

The cost of 51 pounds, at $3\frac{1}{8}$ cents, is $\frac{1}{3}$ of \$5.10, or \$1.70.

26. Find the cost of 39 pounds of nails, at $3\frac{1}{8}$ cents.

27. Find the cost of 42 pounds of nails, at $3\frac{1}{8}$ cents.

28. Find the cost of 80 horses, at \$125 a head.

Solution. — 125 is *one-eighth* of 1000.

The cost of 80 horses, at \$1000, is \$80000.

The cost of 80 horses, at \$125, is $\frac{1}{8}$ of \$80000, or \$10000.

29. Find the cost of 144 cattle, at \$125 a head.

30. Find the cost of 72 cattle, at \$125 a head.

MISCELLANEOUS PROBLEMS.

1. Find the cost of 168 yards of calico, at $12\frac{1}{2}$ cents a yard.

2. Find the cost of 64 yards of muslin, at $12\frac{1}{2}$ cents a yard.

3. Find the cost of 96 yards of sheeting, at $12\frac{1}{2}$ cents a yard.

4. Find the cost of 48 slates, at $12\frac{1}{2}$ cents apiece.

5. Find the cost of 108 books, at $12\frac{1}{2}$ cents apiece.

6. Find the cost of 120 spellers, at $12\frac{1}{2}$ cents apiece.

7. Find the cost of 27 lead-pencils, at $3\frac{1}{2}$ cents apiece.

8. Find the cost of 39 knives, at $33\frac{1}{3}$ cents apiece.
9. Find the cost of 96 readers, at $33\frac{1}{3}$ cents apiece.
10. Find the cost of 96 pen-holders, at $6\frac{1}{4}$ cents apiece.
11. Find the cost of 80 pen-holders, at $6\frac{1}{4}$ cents apiece.

CANCELLATION.

100. Cancellation is often employed to shorten the work when the product of several numbers is to be divided by the product of several numbers. It consists in applying Principle Sixth, Art. 82.

1. Divide the product of $18 \times 20 \times 33$ by $12 \times 15 \times 11$.

FACTORS OF DIVIDEND.

- 18 equals $2 \times 3 \times 3$.
 20 equals $2 \times 2 \times 5$.
 33 equals 3×11 .

FACTORS OF DIVISOR.

- 12 equals $2 \times 2 \times 3$.
 15 equals 3×5 .
 11 equals 11.

OPERATION.

$$\begin{array}{r} \text{Dividend, } 18 \times 20 \times 33 \\ \text{Divisor, } 12 \times 15 \times 11 \end{array}$$

Explanation. — The factors 2, 2, 3, 3, 5, and 11, are common to both dividend and divisor, and may therefore be rejected, by Sixth Principle, Art. 82, without changing the quotient. The remaining factors in the dividend are 2 and 3. Hence, 2 times 3, or 6, is the quotient.

2. Let the pupils make and solve ten similar examples.

NOTE. — For further exercises in Cancellation, see close of promiscuous problems.

PROMISCUOUS PROBLEMS.

1. A man bought three farms. The first contained 113 acres, and cost \$87 an acre ; the second contained 78 acres, and cost \$91 an acre ; the third contained 89

acres, and cost \$31 an acre. How many acres did the three farms contain, and how much did they all cost?

2. A merchant purchased a lot of flour, consisting of 814 barrels, at \$9 a barrel; a second lot consisting of 148 barrels, at \$13 a barrel. How many more barrels of the second lot, and at the same price, could he have bought for the price paid for the first lot?

3. The distance from the town A to the town B, lying directly south of A, is 10 miles greater than the distance from A to C, a town lying directly north of A. It takes 2 hours longer to travel from A to B than from A to C. What is the distance from B to C, if it takes 5 hours, at the same rate, to travel it?

4. How much more must you pay for 37 cords of wood, at \$7 a cord, than for 48 barrels of flour, at \$5 a barrel?

5. A boy went to market to buy apples, peaches, and potatoes. He bought a peck of apples for 32 cents, 3 pecks of peaches for \$1.37, and 1 bushel of potatoes for 90 cents. He gave the market-man a five-dollar bill, and received in change \$2.41. Was this correct? How great was the mistake? In whose favor?

6. Find the cost of 112 bushels of potatoes, at \$1.13 a bushel.

7. Find the cost of 11 bushels of apples, at \$1.46 a bushel.

8. Find the cost of 17 horses, at \$118 apiece.

9. Find the cost of 71 sheep, at \$8.11 apiece.

10. Find the cost of 19 cows, at \$19 apiece.

11. Find the Least Common Multiple of 28, 49, 63, 91.

12. Find the Least Common Multiple of 42, 52, 62, 72.

13. Find the Least Common Multiple of 81, 128, 256, 625.

14. Find the Greatest Common Divisor of 356, 898, and 1254.

15. Find the Greatest Common Divisor of 1126, 1296, and 1548.

16. Find the Greatest Common Divisor of 2882, 4097, 4579.

17. Prove that 2116 is the difference between 809 and 2925.

18. The difference between two numbers is 11126, and the less of the two numbers is 4347. Find the greater number.

19. The difference between two numbers is 9168, and one of these two numbers is 3123. Find the other number.

20. When the difference of two numbers is known, and one of the numbers, how is the other number found?

21. What fundamental operation is involved in the preceding question?

22. The less of two numbers is 500, and the greater number is 1125. What is their sum?

23. The less of two numbers is 105, and the greater number is 1511. What is their difference?

24. The less of two numbers is 827, and their sum is 2472. Find the greater number.

25. The less of two numbers is 287, and their difference is 215. Find the greater number.

26. The greater of two numbers is 2941, and the less number is 1116. Find their difference.

27. The greater of two numbers is 9214, and their sum is 9241. Find the less number.

28. Find the product of 2222 by 222.

29. The product of two numbers is 4579, and one of those numbers is 241. Find the other number.

30. When the product of several numbers is known, and all of the numbers but one, how is the remaining number found?

31. What number multiplied by 24 times 72 will produce the number 68200?

32. To what term in division does the term product correspond?

33. To what term in division does the term multiplier in multiplication correspond?

34. When the product and the multiplier are known, how is the multiplicand found?

35. The product of two numbers is 365092, and the multiplier is 767. Find the multiplicand.

36. How is Addition proved?

37. How is Subtraction proved?

38. How is Multiplication proved?

39. How is Division proved when there is no remainder?

40. How is Division proved when there is a remainder?

41. Make five examples in Addition, and prove the work.

42. Make five examples in Subtraction, and prove the work.

43. Make five examples in Multiplication, and prove the work.

44. Make five examples in Division, and prove the work.

45. State the distinction between the Greatest Common Divisor of two or more numbers, and their Least Common Multiple.

46. How can you multiply by 10, 100, 1000, and so on? Why?

47. How can you divide by 10, 100, 1000, and so on? Why?

48. Find the product of 378 multiplied by 63, using the factors of the multiplier.

49. Find the cost of one sack of coffee, when 237 sacks cost \$20145.

50. If you can travel 27 miles in one day, how many miles can you travel in 79 days?

51. If 1578 barrels of pork cost \$23670, what did 1 barrel cost?

52. If you wished to exchange 25 pounds of butter for 45 pounds of sugar, at 8 cents per pound, what must be the price of butter? Ans. $14\frac{2}{3}$ cents.

53. If \$24 will pay for 8 days' work, what is 1 day's work worth?

54. If 8 men earn \$32, how much does each man earn?

55. If \$48 will buy 12 cords of wood, what is the cost of 1 cord?

56. If 8 barrels of flour are worth \$56, what is 1 barrel worth?

57. If 8 men can earn \$56, what can 1 man earn?

58. If 8 men can do a piece of work in 56 days, in how many days can 1 man do the same work?

59. What is the difference between examples 57 and 58?

60. Can 1 man earn, in a given time, more, or less, money than 8 men can earn?

61. Can 1 man do a certain piece of work in a greater, or a less, number of days than 8 men can do it?

62. Find the time in which 5 men can do the same piece of work that 10 men can do in 20 days.

63. Find the time in which 10 men can do the same piece of work that 5 men can do in 20 days.

64. How many men are required to earn as much in 5 days, as 10 men can earn in 20 days?

65. How many men are required to earn as much in 10 days, as 5 men earn in 20 days?

66. A man bought 7 horses at \$96.50 each; 9 oxen at \$31.75 each; and 43 cows at \$14.25 each. What did they all cost him?

67. Add \$127.32, \$415.12, \$117.10, \$816.02, \$2.27, and \$66.76.

68. How much more will 12 acres of land cost, at \$71 an acre, than 15 acres, at \$43 an acre?

69. A merchant bought 35 yards of calico, at 9 cents a yard, and paid for it in sugar at 15 cents a pound. How many pounds of sugar did it take?

70. A farmer sold 30 dozen eggs, at $16\frac{2}{3}$ cents a dozen,

and 32 pounds of butter, at $12\frac{1}{2}$ cents a pound. He bought 48 yards of cloth, at $6\frac{1}{2}$ cents a yard. How much money had he remaining?

71. How much will it cost to plow 7 acres of land, at the rate of \$1.75 per acre?

72. A farmer raised 28 bushels of wheat per acre on 42 acres of ground, and 42 bushels of corn per acre on 82 acres of ground. If he sold his wheat at \$1.25 per bushel, and his corn at $62\frac{1}{2}$ cents per bushel, what was his crop worth to him?

73. Find the cost of 156 reams of paper, if 75 reams cost \$243.75.

74. Find the cost of 1 box of oranges, when 610 boxes cost \$268.40.

75. If each of five horses costs \$112.12, and two of them were again sold for \$142.18, how much was gained?

76. What will 13 acres of land cost, if 395 acres cost \$186.045?

77. Find the cost of 29 acres, at \$36.36 per acre.

78. Find the cost of 17 pairs of skates, at \$1.37 a pair.

79. Find the cost of 311 cords of wood, at \$7.12 a cord.

80. Find the cost of 914 kegs of butter, each 56 lbs., at $12\frac{1}{2}$ cents per pound.

81. Find the cost of 96 pounds of beef, at $6\frac{1}{4}$ cents per pound.

82. Find the sum of \$171.71, \$711.17, \$804.05, and \$206.07.

83. How much more will 18 horses cost, at \$91 a head, than 32 mules, at \$45.12 a head?

84. If 117 cattle cost \$14625, what will one animal cost?

85. Find a number which will exactly contain the several numbers, 27, 36, 42, 56, 68, and 91.

86. What number will exactly divide the several numbers 297, 351, 405, 486, 567, and 648?

87. Find the cost of 87 pounds of lard, at 9 cents per pound.

88. Find the cost of 187 pounds of butter, at 15 cents per pound.

89. Find the cost of 17 pounds of cheese, at 9 cents a pound.

90. Find the cost of 12 dozen eggs, at 11 cents a dozen.

91. Find the cost of 19 pounds of starch, at 7 cents a pound.

92. Find the cost of 35 pounds of nails, at 6 cents a pound.

93. Find the cost of 53 pounds of coffee, at 34 cents a pound.

94. Find the cost of 29 pounds of tea, at \$1.12 cents a pound.

95. Find the cost of 37 pounds of raisins, at 27 cents a pound.

96. Find the cost of 73 pounds of soap, at 11 cents a pound.

97. Find the cost of 33 quarts of milk, at 4 cents a quart.

98. Find the cost of 77 quarts of cherries, at 9 cents a quart.

99. Find the cost of 92 brooms, at 21 cents apiece.

100. Find the cost of 21 hats, at \$1.87 apiece.

101. Find the cost of 18 cords of wood, at \$4.75 a cord.

102. Find the cost of 81 cords of wood, at \$7.45 a cord.

103. Find the cost of 74 acres of ground, at \$37.50 an acre.

104. Find the cost of 12 farms, at \$1127.91 apiece.

105. Find the cost of 27 suits of clothes, at \$11.25 apiece.

106. Find the cost of 72 overcoats, at \$17.25 apiece.

107. Mr. A travels at the rate of 12 miles in 2 hours, and Mr. B travels at the rate of 15 miles in 5 hours. How many miles does A gain on B in 1 hour?

108. If A and B should travel at these rates for 10 hours, how much farther would A go than B?

109. In the above example, the rates of travel remaining the same, how long would it take A to overtake B, who is 60 miles ahead?

110. If one of two trains has 100 miles the start, and moves at the rate of 15 miles an hour, how long will a second train, running at the rate of 25 miles an hour, be in overtaking it?

111. A man divided 117 cents among 9 boys, giving to each the same number of cents; how many cents did each boy get?

112. A man divided 192 marbles among a certain lot

of boys, giving to each boy the same number of marbles; how many boys were there?

113. Find the number of cents required to pay 18 boys, if each boy is paid 18 cents.

114. If 17 men are paid \$187 for working a certain number of days, how many days did they work?

115. How much can 28 men earn in 29 days, at \$4 per day? In what time can 7 boys earn the same amount at \$2 per day?

116. At the rate of \$51 for 17 days, in how many days can 3 men earn \$81?

117. At the rate of \$42 for 14 days, how many men can earn \$96 in two days?

118. How many men can earn \$192 in 7 days, if 7 men earn \$14 in 2 days?

Suggestion. — By the last condition, how much does 1 man earn in 2 days? How much in one day?

Then, by the first condition, how much does 1 man earn in 7 days?

Finally, how many men will be required to earn \$192 in 7 days?

119. If 1 man can earn \$4 in one week, how many dollars can he earn in 10 weeks?

120. If \$1 earns 4 cents in one year, how many cents will it earn in 10 years?

121. If the use of \$1 for one year is worth five cents, how many cents is its use worth for 10 years?

122. If the interest on \$1 for one year is 6 cents, what is the interest on \$1 for 10 years?

123. If one man earns \$5 in 1 week, how many dollars can 6 men earn in 10 weeks?

124. If \$1 earns 6 cents in one year, how many cents will \$6 earn in 10 years?

125. If the use of \$1 for one year is worth 8 cents, how many cents is the use of \$12 worth for 12 years?

126. If the interest on \$1 for one year is 8 cents, what is the interest on \$12 for 12 years?

127. Six men earn \$12 in 1 week; how much does one man earn in one week? How much in 3 weeks? How much do 3 men earn in one week? How much do 3 men earn in 4 weeks?

128. If \$6 earn 12 cents in one year, how many cents does \$1 earn in one year? How many cents in 3 years? How many cents do \$5 earn in one year? How many cents do \$5 earn in 12 years?

129. If the use of \$6 for one year is worth 12 cents, how many cents is the use of \$18 worth for one year? How many cents is the use of \$18 worth for 12 years?

130. Find the interest on \$20 for 5 years, if the interest on \$7 for 3 years is 84 cents.

131. How long will it take 5 men to earn \$40, if 1 man can earn \$2 in 1 day?

132. In how many years will \$5 earn 40 cents, if \$1 earns 2 cents in one year?

133. In how many years will \$7 earn 21 cents, if \$1 earns 3 cents in one year?

134. In how many days can 7 men earn \$84, if 3 men earn \$24 in 2 days?

135. In how many years will \$7 earn 84 cents, if \$3 earn 24 cents in 2 years?

136. Find the Greatest Common Divisor of 1728, 1844, and 2026.

137. Find the Least Common Multiple of 108, 208, 828, and 882, by factoring.

138. Find the Least Common Multiple of 8, 12, 15, 18, 21, 27, 32, 42, 51, and 91.

139. If you multiply any number by 4, and then divide the product thus found by 2, will the result be greater, or less, than the original number?

140. If you multiply any number by 8, and divide the product thus found by 7, will the result be greater, or less, than the original number?

141. Find the cost of 32 dozen eggs, at $12\frac{1}{2}$ cents a dozen.

142. Find the cost of 42 arithmetics, at $16\frac{2}{3}$ cents apiece.

143. Find the cost of 112 arithmetics, at $16\frac{2}{3}$ cents apiece.

144. Find the cost of 10 grammars, at 20 cents apiece.

145. Find the cost of 12 grammars, at 25 cents apiece.

146. Find the cost of 30 philosophies, at $33\frac{1}{3}$ cents apiece.

147. Find the cost of 88 horses, at \$125 a head.

148. Find the cost of 64 horses, at \$125 a head.

Use cancellation in the following :

149. Multiply 24, 36, 48, and divide by $18 \times 12 \times 6$.

150. Divide $56 \times 65 \times 81$ by $27 \times 13 \times 8 \times 5$.

151. Divide $100 \times 96 \times 84 \times 48$ by $24 \times 14 \times 16 \times 25$.

152. Divide $250 \times 1000 \times 125$ by $8 \times 40 \times 50$.

COMMON FRACTIONS.

101. A **unit** or any single thing, or several units considered as a **single thing**, may be **divided** into any number of **equal parts**.

102. When any thing is divided into **two** equal parts, each part is **one-half** of the whole thing, and is called a **fractional part** of the whole. This **fraction** is expressed thus: $\frac{1}{2}$. The **whole thing** thus divided is expressed thus: $\frac{2}{2}$, which is equal to **1 unit**.

How many halves are there in 1 unit?

How many halves are there in 2 units?

How many halves are there in 3 units?

How many halves are there in 10 units?

103. When any thing is divided into **three** equal parts, each part is **one-third** of the whole thing, and **two** of these equal parts are **two-thirds** of the whole thing. These **fractions** are expressed thus: $\frac{1}{3}$, $\frac{2}{3}$. The **whole thing** thus divided is written thus: $\frac{3}{3}$, which is equal to **1 unit**.

How many thirds are there in 1 unit?

How many thirds are there in 2 units?

How many thirds are there in 3 units?

How many thirds are there in 10 units?

104. When any thing is divided into **four** equal parts, each part is **one-fourth** of the whole thing. **Two** of these equal parts are **two-fourths** of the whole thing. **Three** of these equal parts are **three-fourths** of the whole thing.

These **fractions** are expressed thus: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$. The **whole thing** thus divided is written thus: $\frac{4}{4}$, which is equal to **1 unit**.

How many fourths are there in 1 unit?

How many fourths are there in 2 units?

How many fourths are there in 3 units?

How many fourths are there in 10 units?

105. A **fraction** is an expression of one or more than one of the equal parts into which any thing is divided.

There are two kinds of **fractions**.

a. **Common Fractions.**

b. **Decimal Fractions.**

106. A **common fraction** is a fraction which has both of its terms expressed.

107. The **terms** of a fraction are its **numerator** and **denominator**.

108. The **denominator** of a **fraction** is the **number** written below the line. It shows the **number of equal parts** the unit is divided into.

109. The **numerator** of a **fraction** is the **number** written above the line. It shows the **number of parts** considered.

110. A **decimal fraction** is a **fraction** whose **denominator** is **10** or some power of **10**, not written, but understood.

There are three classes of **common fractions**.

a. **Simple fractions**; as, $\frac{1}{2}$, $\frac{2}{8}$, $\frac{3}{4}$.

b. **Compound fractions**; as, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

c. **Complex fractions**; as, $\frac{\frac{1}{2}}{\frac{2}{3}}$, $\frac{\frac{21}{8}}{\frac{3}{4}}$, $\frac{3\frac{1}{2}}{4\frac{1}{8}}$.

111. A **simple fraction** is a fraction both terms of which are integers ; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$.

112. A **compound fraction** is a fraction of a fraction ; as, $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$.

113. A **complex fraction** is a fraction which has a fraction in one or both of its terms ; as, $\frac{3}{2\frac{1}{2}}$, $\frac{\frac{1}{2}}{\frac{2}{3}}$, $\frac{3\frac{1}{2}}{5}$.

114. There are two classes of **simple fractions**.

a. **Proper fractions** ; as, $\frac{4}{5}$, $\frac{7}{8}$.

b. **Improper fractions** ; as, $\frac{4}{4}$, $\frac{5}{3}$.

115. A **proper fraction** is a fraction whose **numerator** is less than its **denominator** ; as, $\frac{2}{3}$, $\frac{4}{5}$.

116. An **improper fraction** is a fraction whose **numerator** is equal to, or **greater** than, its **denominator** ; as, $\frac{4}{4}$, $\frac{8}{7}$.

117. A **mixed number** is a **whole number** and a **fraction** united in a single expression ; as, $2\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{8}$.

EXERCISES IN FRACTIONS.

1. What is $\frac{1}{2}$ of 1?

6. What is $\frac{2}{3}$ of 2?

2. What is $\frac{1}{3}$ of 2?

7. What is $\frac{3}{4}$ of 3?

3. What is $\frac{1}{4}$ of 3?

8. What is $\frac{4}{5}$ of 4?

4. What is $\frac{1}{5}$ of 4?

9. What is $\frac{5}{6}$ of 5?

5. What is $\frac{1}{6}$ of 5?

10. What is $\frac{6}{7}$ of 6?

Analysis of Example 6. — Since $\frac{1}{3}$ of 2 is the same as $\frac{2}{3}$ of 1, $\frac{2}{3}$ of 2 must be twice $\frac{2}{3}$ of 1, which is $\frac{4}{3}$ of 1.

118. To reduce a whole or mixed number to an **improper fraction**.

1. How many thirds in 1 unit?

2. How many thirds in $1\frac{1}{3}$ units?

3. How many thirds in $1\frac{2}{3}$ units?

4. How many thirds in $2\frac{1}{3}$ units?

5. How many thirds in $3\frac{2}{3}$ units?

Analysis of Example 5.—Since there are $\frac{1}{3}$ in 1 unit, there must be 3 times $\frac{1}{3}$, or $\frac{2}{3}$ in 2 units, and in $3\frac{2}{3}$ there must be $\frac{2}{3} + \frac{2}{3}$, or $\frac{4}{3}$.

Write a rule for this case.

Reduce the following to improper fractions :

- | | | | |
|---------------------|---------------------|----------------------|-----------------------|
| 1. $2\frac{1}{2}$. | 5. $6\frac{1}{8}$. | 9. $2\frac{1}{4}$. | 13. $4\frac{1}{5}$. |
| 2. $3\frac{1}{2}$. | 6. $7\frac{2}{3}$. | 10. $3\frac{3}{4}$. | 14. $5\frac{2}{5}$. |
| 3. $4\frac{1}{2}$. | 7. $8\frac{1}{8}$. | 11. $4\frac{3}{4}$. | 15. $6\frac{3}{5}$. |
| 4. $5\frac{1}{2}$. | 8. $9\frac{2}{8}$. | 12. $5\frac{1}{4}$. | 16. $10\frac{4}{5}$. |

119. To find a fractional part of a number.

- Find $\frac{1}{2}$ of 2 ; of 4 ; of 6 ; of 8 ; of 10 ; of 16.
- Find $\frac{1}{3}$ of 3 ; of 6 ; of 9 ; of 12 ; of 18 ; of 15.
- Find $\frac{1}{4}$ of 4 ; of 8 ; of 12 ; of 20 ; of 16 ; of 24.
- Find $\frac{2}{3}$ of 3 ; of 9 ; of 6 ; of 15 ; of 18 ; of 12.
- Find $\frac{3}{4}$ of 24 ; of 16 ; of 20 ; of 12 ; of 8 ; of 40.

120. To find a number from one or more of its fractional parts.

- | | |
|--|----------|
| 1. 2 is $\frac{1}{2}$ of what number? | Ans. 4. |
| 2. 3 is $\frac{1}{3}$ of what number? | Ans. 9. |
| 3. 4 is $\frac{1}{4}$ of what number? | Ans. 16. |
| 4. 5 is $\frac{1}{5}$ of what number? | Ans. 25. |
| 5. 4 is $\frac{2}{3}$ of what number? | Ans. 6. |
| 6. 10 is $\frac{2}{3}$ of what number? | Ans. 15. |

Analysis of Example 5.—If 4 is $\frac{1}{3}$ of some number, $\frac{1}{3}$ of that number must be $\frac{1}{3}$ of 4, which is 2. Since 2 is $\frac{1}{3}$ of the number required, the whole of that number is 3 times 2, which is 6.

7. 12 is $\frac{3}{4}$ of what number? Ans. 16.
 8. 12 is $\frac{2}{3}$ of what number?
 9. 12 is $\frac{6}{7}$ of what number?
 10. $\frac{1}{2}$ of 12 is $\frac{1}{8}$ of what number?
 11. $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of what number? Ans. 10.

Suggestion.—The above and all similar examples afford an excellent drill for the purpose of fixing in the mind of the pupil the various logical relations which numbers sustain to each other. Giving the abstract answer in questions similar to the above, is an excellent test of a pupil's knowledge of these relations. Thus, in the eleventh example, $\frac{2}{3}$ of 12 is $\frac{4}{5}$ of that number $\frac{1}{5}$ of which is $\frac{1}{4}$ of 2 times $\frac{1}{5}$ of 12.

12. $\frac{4}{5}$ of 10 is $\frac{2}{3}$ of how many times 3?
 13. $\frac{3}{7}$ of 21 is $\frac{5}{6}$ of how many times 5?
 14. $\frac{3}{4}$ of 20 is $\frac{5}{8}$ of how many times $\frac{2}{3}$ of 9? Ans. 3.
 15. $\frac{4}{5}$ of 25 is $\frac{5}{6}$ of how many times $\frac{1}{2}$ of 12?
 16. $\frac{5}{6}$ of $\frac{3}{4}$ of 24 is $\frac{8}{11}$ of how many times $\frac{2}{3}$ of 33?

121. To change halves to fourths.

1. How many fourths in $\frac{1}{2}$?

Analysis.—In 1 unit there are $\frac{1}{2}$. Hence, in $\frac{1}{2}$ a unit there are only $\frac{1}{4}$.

2. How many fourths in $\frac{3}{2}$? In $\frac{7}{2}$? In $\frac{9}{2}$?
 3. How many fourths in $\frac{5}{2}$? In $\frac{6}{2}$? In $\frac{8}{2}$?

122. To change fourths to eighths.

1. How many eighths in $\frac{1}{4}$?

Analysis.—In 1 unit there are $\frac{1}{4}$. Hence, in $\frac{1}{4}$ of a unit there are only $\frac{1}{8}$.

2. How many eighths in $\frac{2}{4}$? In $\frac{3}{4}$? In $\frac{5}{4}$?
 3. How many eighths in $\frac{6}{4}$? In $\frac{7}{4}$? In $\frac{9}{4}$?

123. To change thirds to sixths, or ninths.

1. How many sixths in $\frac{1}{3}$?
2. How many ninths in $\frac{2}{3}$?

ANALYSIS.

- a.* In 1 unit there are $\frac{6}{3}$. Hence, in $\frac{1}{3}$ of a unit there are only $\frac{2}{6}$.
b. In 1 unit there are $\frac{9}{3}$. Hence, in $\frac{1}{3}$ of a unit there are only $\frac{3}{9}$.
 In $\frac{2}{3}$ of a unit there are 2 times $\frac{3}{9}$, which is $\frac{6}{9}$.

Suggestion. — The preceding and similar exercises should be continued until pupils become thoroughly familiar with the process, and with the principle on which the process depends.

124. To find the number of units in any number of halves, thirds, or fourths.

1. How many units in $\frac{10}{2}$?
2. How many units in $\frac{12}{3}$?
3. How many units in $\frac{16}{4}$?

ANALYSIS.

a. In a given number of units there are twice as many halves as units. Hence, in a given number of halves there are only $\frac{1}{2}$ as many units as there are halves. Then, in $\frac{10}{2}$ there are only 5 units.

b. In a given number of units there are three times as many thirds as units. Hence, in a given number of thirds there are only $\frac{1}{3}$ as many units as there are thirds. Then, in $\frac{12}{3}$ there are only 4 units.

c. In a given number of units there are four times as many fourths as there are units. Hence, in a given number of fourths there are only $\frac{1}{4}$ as many units as there are fourths. Then, in $\frac{16}{4}$ there are only 4 units.

Change the following fractions to whole numbers :

- | | | | | |
|---------------------|---------------------|---------------------|---------------------|----------------------|
| 1. $\frac{10}{2}$. | 4. $\frac{12}{3}$. | 7. $\frac{8}{4}$. | 10. $\frac{7}{2}$. | 13. $\frac{11}{2}$. |
| 2. $\frac{14}{2}$. | 5. $\frac{15}{3}$. | 8. $\frac{16}{4}$. | 11. $\frac{8}{3}$. | 14. $\frac{14}{3}$. |
| 3. $\frac{18}{2}$. | 6. $\frac{21}{3}$. | 9. $\frac{20}{4}$. | 12. $\frac{9}{4}$. | 15. $\frac{15}{4}$. |

125. a. *Write a rule for changing whole or mixed numbers to improper fractions.*

b. *Write a rule for changing improper fractions to whole numbers.*

MISCELLANEOUS EXAMPLES IN REVIEW.

1. How many halves in 3 units?
2. How many thirds in 2 units?
3. How many units in 12 halves?
4. How many units in $\frac{15}{8}$?
5. How many twelfths in $\frac{2}{3}$?
6. How many tenths in $\frac{2}{5}$?
7. How many halves in $4\frac{1}{2}$?
8. Change $7\frac{2}{5}$ to an improper fraction.
9. Change $2\frac{1}{7}$ to a whole number.
10. Change $1\frac{7}{8}$ to a mixed number.
11. 9 is $\frac{1}{2}$ of what number?
12. 8 is $\frac{2}{3}$ of what number?
13. $\frac{4}{5}$ of 25 is $\frac{1}{12}$ of how many times $\frac{2}{3}$ of 6?
14. In 1 apple how many fourths of an apple?
15. How many thirds of an apple in 3 apples?
16. In $\frac{10}{3}$ of an orange how many oranges?
17. In $\frac{3}{4}$ of an apple how many twelfths of an apple?
18. Three boys share 6 oranges equally. How many oranges does each boy get? How many do 2 boys get?
19. A barrel of flour is divided equally among 7 persons. What part of the whole do 5 persons get?
20. If 5 men earn a certain sum of money, what part of the whole sum do 3 men earn?
21. $11\frac{1}{2}$ is $\frac{1}{2}$ of what number?

Ans. 23.

22. Find $\frac{2}{7}$ of 14 ; of 21 ; of 28 ; of $4\frac{1}{2}$.

23. A farmer had 65 bushels of wheat, and sold $\frac{3}{5}$ of it ; how many bushels had he left? Ans. 26.

24. A has $\frac{5}{6}$ as many dollars as B. If A has \$15, how much more money has B than A?

25. John and James have together 78 cents ; $\frac{2}{3}$ of their money is 4 times the number of cents John has ; how many more cents has James than John?

Ans. 52 cents.

Note. — *A thorough knowledge of the following six principles is necessary to a thorough knowledge of fractions.*

First Principle.

126. *Multiplying the numerator of a fraction by any whole number multiplies the value of the fraction as many times as there are units in the multiplier.*

DEMONSTRATION.

Multiplying the numerator of a fraction increases the number of parts taken, while the size of those parts expressed by the denominator remains unchanged.

Second Principle.

127. *Dividing the numerator of a fraction by a whole number divides the value of the fraction as many times as there are units in the divisor.*

DEMONSTRATION.

Dividing the numerator of a fraction reduces the number of parts taken, while the size of those parts expressed by the denominator remains unchanged.

Third Principle.

128. *Multiplying the denominator of a fraction by a whole number divides the value of the fraction as many times as there are units in the multiplier.*

DEMONSTRATION.

Multiplying the denominator of a fraction diminishes the size of the parts, while the number of parts expressed by the numerator remains unchanged.

Fourth Principle.

129. *Dividing the denominator of a fraction by a whole number multiplies the value of the fraction as many times as there are units in the divisor.*

DEMONSTRATION.

Dividing the denominator of a fraction increases the size of the parts, while the number of parts expressed by the numerator remains unchanged.

Fifth Principle.

130. *Multiplying both terms of a fraction by the same number does not change its value.*

DEMONSTRATION.

Multiplying the numerator multiplies the value of the fraction as many times as multiplying the denominator divides the fraction. Hence, the value of the fraction remains unchanged.

Sixth Principle.

131. *Dividing both terms of a fraction by the same number does not change its value.*

DEMONSTRATION.

Dividing the numerator divides the value of the fraction as many times as dividing the denominator multiplies it. Hence, the value of the fraction remains unchanged.

REVIEW.

1. What effect on the value of a fraction has multiplying its numerator? Why?
2. What effect on the value of a fraction has multiplying the denominator? Why?
3. What effect on the value of a fraction has dividing the numerator? Why?
4. What effect on the value of a fraction has dividing the denominator? Why?
5. What effect on the value of a fraction has multiplying both terms by the same number? Why?
6. What effect on the value of a fraction has dividing both terms by the same number? Why?

EXAMPLES.

Multiply the numerator of each of the following fractions by 2, and explain the result :

1. $\frac{3}{4}$.

4. $\frac{6}{7}$.

7. $\frac{2}{5}$.

10. $\frac{4}{9}$.

2. $\frac{4}{5}$.

5. $\frac{7}{8}$.

8. $\frac{3}{8}$.

11. $\frac{4}{11}$.

3. $\frac{5}{6}$.

6. $\frac{8}{9}$.

9. $\frac{4}{7}$.

12. $\frac{5}{12}$.

Multiply the denominator of each of the following fractions by 2, and explain the result :

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. $\frac{2}{3}$. | 4. $\frac{3}{2}$. | 7. $\frac{5}{6}$. | 10. $\frac{6}{5}$. |
| 2. $\frac{3}{4}$. | 5. $\frac{4}{3}$. | 8. $\frac{6}{7}$. | 11. $\frac{7}{6}$. |
| 3. $\frac{4}{5}$. | 6. $\frac{5}{4}$. | 9. $\frac{7}{8}$. | 12. $\frac{8}{7}$. |

Divide the numerator of each of the following fractions by 3, and explain the result :

- | | | | |
|---------------------|---------------------|--------------------|-----------------------|
| 1. $\frac{3}{8}$. | 4. $\frac{6}{8}$. | 7. $\frac{3}{4}$. | 10. $\frac{15}{20}$. |
| 2. $\frac{6}{9}$. | 5. $\frac{9}{6}$. | 8. $\frac{3}{6}$. | 11. $\frac{18}{20}$. |
| 3. $\frac{9}{12}$. | 6. $\frac{12}{9}$. | 9. $\frac{3}{7}$. | 12. $\frac{21}{20}$. |

Divide the denominator of each of the following fractions by 4, and explain the result :

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| 1. $\frac{5}{4}$. | 4. $\frac{16}{12}$. | 7. $\frac{24}{20}$. | 10. $\frac{5}{36}$. |
| 2. $\frac{8}{12}$. | 5. $\frac{12}{8}$. | 8. $\frac{30}{28}$. | 11. $\frac{7}{36}$. |
| 3. $\frac{12}{16}$. | 6. $\frac{15}{20}$. | 9. $\frac{10}{32}$. | 12. $\frac{9}{44}$. |

Multiply both terms of each of the following fractions by 5, and explain the result :

- | | | | |
|--------------------|--------------------|--------------------|---------------------|
| 1. $\frac{2}{7}$. | 4. $\frac{7}{8}$. | 7. $\frac{2}{8}$. | 10. $\frac{3}{4}$. |
| 2. $\frac{7}{2}$. | 5. $\frac{4}{7}$. | 8. $\frac{3}{2}$. | 11. $\frac{4}{5}$. |
| 3. $\frac{3}{7}$. | 6. $\frac{7}{4}$. | 9. $\frac{4}{8}$. | 12. $\frac{5}{6}$. |

Divide both terms of each of the following fractions by 3, and explain the result :

- | | | | |
|--------------------|----------------------|----------------------|-----------------------|
| 1. $\frac{3}{9}$. | 4. $\frac{12}{15}$. | 7. $\frac{21}{24}$. | 10. $\frac{30}{9}$. |
| 2. $\frac{6}{9}$. | 5. $\frac{15}{12}$. | 8. $\frac{24}{18}$. | 11. $\frac{33}{15}$. |
| 3. $\frac{9}{6}$. | 6. $\frac{18}{21}$. | 9. $\frac{27}{21}$. | 12. $\frac{36}{12}$. |

132. The first and fourth principles furnish the rule for multiplying a fraction by an integer.

The second and third principles furnish the rule for dividing a fraction by an integer.

The fifth principle furnishes the rule for **changing a given fraction to an equivalent fraction of a given denominator.**

The fifth principle also furnishes a rule for **reducing a given fraction to lower terms.**

The sixth principle furnishes the rule for **changing complex to simple fractions.**

REDUCTION OF FRACTIONS.

133. Reduction of fractions consists in changing their form without changing their value.

- a. *Fractions may be reduced to lower terms.*
- b. *Compound fractions may be reduced to simple ones.*
- c. *A whole or mixed number may be reduced to an improper fraction.*
- d. *Improper fractions may be reduced to whole or mixed numbers.*
- e. *Fractions may be reduced to equivalent fractions having a given denominator.*
- f. *Complex fractions may be reduced to simple ones.*
- g. *Fractions may be reduced to equivalent fractions having a common denominator.*

CASE I.

134. A fraction is in its lowest terms when its numerator and denominator are prime to each other; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{15}{16}$.

135. When the terms of a fraction have a common factor, the fraction may be reduced to lower terms by cancelling the common factor. (Sixth Principle.)

136. To reduce a fraction to its lowest terms.

Example. — Reduce $\frac{91}{169}$ to its lowest terms.

Solution. — $\frac{91}{169} = \frac{7 \times 13}{13 \times 13}$; cancelling the common factor 13, we get $\frac{7}{13}$ as the value of $\frac{91}{169}$ reduced to its lowest terms.

RULE. — *Divide both terms of the fraction by their greatest common divisor.* (Sixth Principle.)

Reduce the following fractions to their lowest terms :

- | | | |
|----------------------|-----------------------|-------------------------------|
| 1. $\frac{2}{4}$. | 7. $\frac{5}{15}$. | 13. $\frac{120}{880}$. |
| 2. $\frac{3}{6}$. | 8. $\frac{5}{10}$. | 14. $\frac{19}{57}$. |
| 3. $\frac{6}{18}$. | 9. $\frac{10}{15}$. | 15. $\frac{451}{1353}$. |
| 4. $\frac{9}{18}$. | 10. $\frac{5}{20}$. | 16. $\frac{333}{777}$. |
| 5. $\frac{12}{18}$. | 11. $\frac{10}{20}$. | 17. $\frac{171492}{42876}$. |
| 6. $\frac{8}{18}$. | 12. $\frac{15}{20}$. | 18. $\frac{113950}{124740}$. |

Note. — Many operations in fractions may be greatly simplified by first reducing the fractions to their lowest terms. This reduction may usually be effected by simple inspection.

CASE II.

137. To reduce compound fractions to simple ones.

Example. — Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.

Solution. — First. $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$.

Second. $\frac{2}{3}$ of $\frac{1}{4} = 2$ times $\frac{1}{12} = \frac{2}{12}$.

Third. $\frac{3}{4}$ of $\frac{1}{4} = 3$ times $\frac{1}{12} = \frac{3}{12}$.

RULE. — *Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

Example. — Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction.

Solution. — The product of the numerators is $2 \times 3 \times 5 = 30$.

The product of the denominators is $3 \times 4 \times 6 = 72$.

Therefore, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6} = \frac{30}{72}$, or $\frac{5}{12}$.

Reduce the following compound fractions to simple ones, and reduce to lowest terms :

1. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

8. $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$.

2. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

9. $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$.

3. $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$.

10. $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$.

4. $\frac{1}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$.

11. $\frac{21}{27}$ of $\frac{18}{42}$ of $\frac{96}{98}$.

5. $\frac{1}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$.

12. $\frac{104}{91}$ of $\frac{51}{68}$ of $\frac{39}{65}$ of $\frac{26}{52}$.

6. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

13. $\frac{2}{7}$ of $\frac{3}{5}$ of $\frac{4}{9}$ of $\frac{9}{15}$ of $\frac{4}{8}$.

7. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$.

14. $\frac{18}{54}$ of $\frac{72}{90}$ of $\frac{27}{88}$ of $\frac{5}{81}$.

15. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{11}$ of $\frac{14}{17}$.

16. John has $\frac{3}{7}$, and James has $\frac{2}{3}$ as much ; what part of a dollar has James?

17. An orange cost $\frac{3}{5}$ of a dime ; how many cents are $\frac{3}{4}$ of an orange worth?

18. Moses had $\frac{2}{5}$ of $\frac{1}{2}$ of \$1, and his brother had $\frac{1}{4}$ as much money ; how many cents had his brother?

19. If a hat is worth \$5 $\frac{1}{2}$, and a pair of shoes are worth $\frac{2}{3}$ as much as the hat, what is the value of the shoes?

20. A carriage cost $\frac{7}{8}$ of \$100, and a horse $\frac{2}{3}$ as much, what was the value of the horse?

CASE III.

138. To reduce a whole or mixed number to an improper fraction.

RULE. — *Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the result over the denominator. (Art. 118.)*

Reduce the following whole or mixed numbers to improper fractions :

1. Reduce $2\frac{1}{2}$ to halves.
2. Reduce $3\frac{1}{3}$ to thirds.
3. Reduce $7\frac{1}{4}$ to fourths.
4. Reduce $13\frac{2}{5}$ to fifths.
5. Reduce $101\frac{5}{9}$ to ninths.
6. How many twelfths in $27\frac{1}{12}$?
7. How many twelfths in $201\frac{2}{12}$?
8. How many fifteenths in 15?
9. How many sixteenths in 16?
10. How many fifteenths in 16?
11. A has $\$6\frac{2}{3}$, and B has $\frac{1}{2}$ as much. How many thirds of a dollar has B? How many sixths of a dollar?
12. A ball cost $5\frac{1}{2}$ cents, and a top $\frac{1}{2}$ as much. How many fourths of a cent did the top cost? How many halves of a cent?
13. A farm cost $\$1001\frac{1}{5}$, and a house $\frac{1}{2}$ as much. How many fifths, and how many tenths, of a dollar did the house cost?

CASE IV.

139. To reduce an improper fraction to a whole or mixed number.

RULE. — *Divide the numerator by the denominator, and write the remainder, if any, over the denominator.*
(Art. 124.)

Reduce to whole or 'mixed numbers :

- | | | |
|--------------------------|----------------------------------|-------------------------|
| 1. $\frac{39}{8}$. | 5. $\frac{65}{18}$. | 9. $\frac{1728}{144}$. |
| 2. $\frac{24}{12}$. | 6. $\frac{91}{18}$. | 10. $\frac{1728}{27}$. |
| 3. $\frac{12579}{221}$. | 7. $\frac{12746815}{5186472}$. | 11. $\frac{1728}{72}$. |
| 4. $\frac{59678}{122}$. | 8. $\frac{21212121}{12121212}$. | 12. $\frac{1728}{86}$. |

CASE V.

140. To reduce fractions to equivalent fractions having a given denominator.

Example. — Reduce $\frac{7}{8}$ to sixteenths.

Solution. — By Fifth Principle.

$$\frac{7}{8} \times \frac{2}{2} = \frac{14}{16}, \text{ since } 16 \div 8 = 2.$$

RULE. — *Divide the given denominator by the denominator of the given fraction, multiply the numerator by the quotient, and write the result over the given denominator.*

Reduce the following fractions to fourths :

- | | | | | |
|---------------------|---------------------|-----------------------|------------------------|------------------------|
| 1. $\frac{1}{2}$. | 5. $4\frac{1}{2}$. | 9. $12\frac{1}{2}$. | 13. $39\frac{1}{2}$. | 17. $112\frac{1}{2}$. |
| 2. $1\frac{1}{2}$. | 6. $5\frac{1}{2}$. | 10. $15\frac{1}{2}$. | 14. $50\frac{1}{2}$. | 18. $133\frac{1}{2}$. |
| 3. $2\frac{1}{2}$. | 7. $6\frac{1}{2}$. | 11. $18\frac{1}{2}$. | 15. $75\frac{1}{2}$. | 19. $166\frac{1}{2}$. |
| 4. $3\frac{1}{2}$. | 8. $9\frac{1}{2}$. | 12. $20\frac{1}{2}$. | 16. $101\frac{1}{2}$. | 20. $175\frac{1}{2}$. |

Reduce the following fractions to eighths :

- | | | | | |
|--------------------|---------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{1}{2}$. | 5. $1\frac{1}{2}$. | 9. 3. | 13. $9\frac{3}{4}$. | 17. $21\frac{1}{4}$. |
| 2. $\frac{1}{4}$. | 6. $1\frac{1}{4}$. | 10. $4\frac{1}{4}$. | 14. $11\frac{1}{4}$. | 18. $31\frac{3}{4}$. |
| 3. $\frac{2}{4}$. | 7. $1\frac{3}{4}$. | 11. $5\frac{3}{4}$. | 15. $15\frac{3}{4}$. | 19. $41\frac{3}{4}$. |
| 4. $\frac{3}{4}$. | 8. 2. | 12. $6\frac{1}{2}$. | 16. $18\frac{3}{4}$. | 20. $51\frac{1}{4}$. |

Reduce the following fractions to twenty-firsts :

1. $\frac{1}{8}$.	6. $\frac{2}{14}$.	11. $\frac{4}{7}$.	16. $10\frac{2}{14}$.
2. $\frac{1}{7}$.	7. $\frac{4}{14}$.	12. $\frac{12}{14}$.	17. $16\frac{4}{14}$.
3. $\frac{2}{8}$.	8. $\frac{6}{14}$.	13. $\frac{5}{7}$.	18. 33.
4. $\frac{2}{7}$.	9. $\frac{8}{14}$.	14. $\frac{6}{7}$.	19. 44.
5. $\frac{3}{7}$.	10. $\frac{10}{14}$.	15. $\frac{1}{7}$.	20. 55.

CASE VI.

141. To reduce complex fractions to simple ones.

Example. — Reduce $\frac{2\frac{1}{8}}{3\frac{1}{2}}$ to a simple fraction.

Solution. — Every complex fraction may be reduced to a simple one by Principle 5. Hence, $\frac{2\frac{1}{8}}{3\frac{1}{2}}$ is the same as $\frac{2\frac{1}{8}}{3\frac{1}{2}} \times \frac{6}{6} = \frac{14}{21}$, or $\frac{2}{3}$.

RULE. — *Multiply both terms of the complex fraction by the least common multiple of the denominators of the fractions composing its terms.*

Reduce the following complex fractions to simple ones, and to their lowest terms :

1. $\frac{\frac{1}{2}}{\frac{1}{8}}$.	4. $\frac{\frac{2}{8}}{\frac{1}{2}}$.	7. $\frac{2\frac{1}{2}}{3}$.	10. $\frac{3\frac{1}{8}}{2}$.
2. $\frac{\frac{1}{8}}{\frac{1}{2}}$.	5. $\frac{\frac{2}{8}}{\frac{1}{4}}$.	8. $\frac{3}{2\frac{1}{2}}$.	11. $\frac{3\frac{1}{8}}{2\frac{1}{2}}$.
3. $\frac{\frac{1}{2}}{\frac{2}{8}}$.	6. $\frac{\frac{2}{8}}{\frac{3}{4}}$.	9. $\frac{2}{3\frac{1}{2}}$.	12. $\frac{2\frac{1}{2}}{3\frac{1}{8}}$.

$$13. \frac{\frac{21}{4}}{\frac{31}{2}} = \frac{41\frac{1}{2}}{51\frac{1}{2}}$$

$$14. \frac{\frac{62}{8}}{\frac{61}{8}} = \frac{55\frac{5}{8}}{65\frac{5}{8}}$$

$$15. \frac{\frac{123}{8}}{\frac{163}{4}} = \frac{202\frac{2}{5}}{251\frac{1}{4}}$$

$$16. \frac{\frac{251}{4}}{\frac{8}{8}} = \frac{251\frac{1}{4}}{251\frac{1}{4}}$$

$$17. \frac{\frac{8}{8}}{\frac{251}{4}} = \frac{251\frac{1}{4}}{251\frac{1}{4}}$$

$$18. \frac{\frac{100}{2}}{\frac{1}{2}} = \frac{100}{100}$$

CASE VII.

142. To reduce fractions to equivalent fractions having a common denominator.

Example. — Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{12}$ to equivalent fractions having a given denominator.

Solution. — Since 24 is a multiple of the denominators of the given fractions, they may be reduced to equivalent fractions having 24 for their common denominator. Hence, by Principle 5, we find that —

$$\begin{aligned} \frac{1}{2} \times \frac{12}{12} &= \frac{12}{24}, \text{ since } 24 \div 2 = 12. \\ \frac{1}{3} \times \frac{8}{8} &= \frac{8}{24}, \text{ since } 24 \div 3 = 8. \\ \frac{1}{6} \times \frac{4}{4} &= \frac{4}{24}, \text{ since } 24 \div 6 = 4. \\ \frac{1}{8} \times \frac{3}{3} &= \frac{3}{24}, \text{ since } 24 \div 8 = 3. \\ \frac{1}{12} \times \frac{2}{2} &= \frac{2}{24}, \text{ since } 24 \div 12 = 2. \end{aligned}$$

Reduce the following fractions to equivalent fractions having a common denominator:

- $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}.$
- $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}.$
- $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{1}{16}.$
- $\frac{2}{8} \text{ of } \frac{3}{4}, 2\frac{3}{8}, 5\frac{2}{5}, \frac{2}{5} \text{ of } \frac{1}{8} \text{ of } 3\frac{1}{2}.$
- $\frac{5}{9} \text{ of } \frac{3}{8} \text{ of } \frac{1}{15}, \frac{2}{9} \text{ of } \frac{6}{15} \text{ of } \frac{27}{80}.$
- $\frac{24}{88}, \frac{36}{48}, \frac{48}{60}, \frac{60}{72}, \frac{72}{84}, \frac{84}{96}, \frac{96}{108}.$

ADDITION OF FRACTIONS.

143. Addition of fractions consists in finding the sum of two or more fractions.

144. a. To add fractions they must be reduced to equivalent fractions having a common denominator, unless they already have a common denominator.

b. Mixed numbers may be added without being reduced to improper fractions, by adding the integral parts and fractional parts separately.

Example.—a. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{12}$.

Solution.—Since 24 is a multiple of the denominators of the given fractions, they may be reduced to equivalent fractions having 24 for their common denominator. Hence, by Principle 5, we find that—

$$\begin{aligned}\frac{1}{2} \times \frac{12}{12} &= \frac{12}{24}, \text{ since } 24 \div 2 = 12. \\ \frac{1}{3} \times \frac{8}{8} &= \frac{8}{24}, \text{ since } 24 \div 3 = 8. \\ \frac{1}{6} \times \frac{4}{4} &= \frac{4}{24}, \text{ since } 24 \div 6 = 4. \\ \frac{1}{8} \times \frac{3}{3} &= \frac{3}{24}, \text{ since } 24 \div 8 = 3. \\ \frac{1}{12} \times \frac{2}{2} &= \frac{2}{24}, \text{ since } 24 \div 12 = 2. \\ \frac{12}{24} + \frac{8}{24} + \frac{4}{24} + \frac{3}{24} + \frac{2}{24} &= \frac{29}{24}, \text{ or } 1\frac{5}{24}, \text{ Ans.}\end{aligned}$$

RULE.—**a.** Reduce the fractions to equivalent fractions having a common denominator.

b. Add the numerators and write their sum over the common denominator.

Find the sum

1. Of $\frac{3}{5}$, $\frac{4}{8}$, $\frac{3}{4}$, and $\frac{5}{6}$.
2. Of $2\frac{2}{3}$, $3\frac{2}{3}$, $4\frac{3}{4}$, and $3\frac{3}{4}$.
3. Of 5 , $\frac{1}{9}$, $7\frac{1}{3}$, $\frac{5}{6}$, and $27\frac{1}{6}$.
4. Of $21\frac{1}{2}$, $31\frac{1}{3}$, $41\frac{1}{4}$, and $51\frac{1}{5}$.
5. Of $12\frac{2}{3}$, $13\frac{3}{4}$, $14\frac{4}{5}$, $15\frac{5}{6}$, and 16 .
6. Of $202\frac{1}{3}$, $303\frac{1}{2}$, $404\frac{1}{5}$, and $505\frac{1}{4}$.

7. John has $\$ \frac{3}{4}$, James $\frac{1}{2}$ as much, George $\frac{2}{3}$ as much, Henry $\frac{2}{3}$ as much as John. How much money have all together?

8. A man has $20\frac{3}{4}$ acres in one field, $40\frac{1}{2}$ acres in another, $\frac{2}{3}$ as many acres in a third as in the other two; how many has he altogether?

9. One-half of John's money equals $\frac{2}{3}$ of William's, and $\frac{3}{4}$ of William's money equals $\frac{4}{5}$ of Henry's. How many dollars has each, and how many have all, if $\frac{2}{3}$ of John's money is $\$48\frac{4}{5}$?

10. Add $72\frac{2}{5}$ acres, $27\frac{1}{8}$ acres, $32\frac{2}{7}$ acres, $51\frac{2}{3}$ acres, $21\frac{1}{5}$ acres.

SUBTRACTION OF FRACTIONS.

145. Subtraction of fractions consists in finding the difference between two fractions.

146. a. To find the difference between two fractions they must be reduced to equivalent fractions having a common denominator, unless they already have a common denominator.

b. The difference between two mixed numbers may be found without reducing them to improper fractions.

Example. — *a.* From $\frac{1}{2}$ take $\frac{1}{3}$.

Solution. — Since 6 is a multiple of the denominators of the given fractions, they may be reduced to equivalent fractions having 6 for their common denominator. Hence we find by Principle 5:

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}, \text{ since } 6 \div 2 = 3.$$

$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}, \text{ since } 6 \div 3 = 2.$$

$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}. \text{ Ans.}$$

RULE. — a. *Reduce the given fractions to equivalent fractions having a common denominator.*

b. *Find the difference between their numerators, and write the result over the common denominator.*

1. From $\frac{3}{4}$ take $\frac{2}{5}$.
2. From $\frac{2}{5}$ take $\frac{1}{8}$.
3. From $\frac{1}{8}$ take $\frac{2}{15}$.
4. From $\frac{2}{15}$ take $\frac{1}{30}$.
5. From $\frac{1}{30}$ take $\frac{1}{40}$.
6. From $\frac{7}{5}$ take $\frac{7}{8}$.
7. From $\frac{5}{8}$ take $\frac{7}{11}$.
8. From $2\frac{4}{5}$ take $1\frac{7}{8}$.
9. From $15\frac{11}{12}$ take $14\frac{1}{11}$.
10. From $\frac{10}{2}$ take $\frac{8}{3}$.
11. From $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$ take $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{2}{5}$.
12. From $\frac{7}{2}$ of $\frac{9}{11}$ of $\frac{2}{3}$ take $\frac{1}{2}$ of $\frac{1}{8}$ of $\frac{1}{4}$.
13. From $\frac{1}{2}$ of $\frac{1}{5}$ of $\frac{1}{7}$ take $\frac{1}{5}$ of $\frac{1}{9}$ of $\frac{1}{11}$.
14. From $\frac{1}{10}$ of $\frac{1}{8}$ of $\frac{1}{17}$ take $\frac{1}{11}$ of $\frac{1}{14}$ of $\frac{1}{19}$.
15. From $\frac{2}{9}$ of $\frac{9}{15}$ of $\frac{8}{71}$ take $\frac{8}{72}$ of $\frac{7}{11}$ of $\frac{1}{8}$.
16. One boy has $\frac{2}{3}$ of $\frac{3}{4}$ of \$1, and another boy has $\frac{1}{3}$ of $\frac{3}{8}$ of \$1. How much has one more than the other?
17. A has $\frac{2}{5}$ of \$1, B has $\frac{5}{12}$ of \$2, C has $\frac{4}{5}$ of \$1, and D has $\frac{3}{8}$ of $\frac{3}{4}$ of \$3. How many more cents have A and D together than B and C?
18. How much greater is $\frac{8}{9}$ of $\frac{2}{3}$ than $\frac{8}{9}$ of $\frac{2}{3}$?
19. To the difference between $\frac{9}{11}$ and $\frac{9}{10}$ add the difference between $\frac{10}{20}$ and $\frac{10}{30}$.
20. From $\frac{4}{5}$ plus $\frac{5}{10}$ take the difference between $\frac{10}{2}$ and $\frac{20}{3}$.

MULTIPLICATION OF FRACTIONS.

147. Multiplication of fractions consists in repeating a fraction as many times, or parts of a time, as there are units, or parts of a unit, in the multiplier.

- a. A fraction may be multiplied by an integer.
 b. An integer may be multiplied by a fraction.
 c. A fraction may be multiplied by a fraction.

CASE I.

148. To multiply a fraction by an integer.

Principles 1 and 4 give the following

RULE.—a. Multiply the numerator of the fraction by the integer; or,

b. Divide the denominator of the fraction by the integer.

MULTIPLY	MULTIPLY	MULTIPLY
1. $\frac{8}{15}$ by 2.	6. $\frac{42}{50}$ by 7.	11. $\frac{102}{121}$ by 2.
2. $\frac{12}{15}$ by 3.	7. $\frac{42}{50}$ by 14.	12. $\frac{121}{132}$ by 11.
3. $\frac{16}{15}$ by 4.	8. $\frac{42}{50}$ by 21.	13. $\frac{132}{144}$ by 11.
4. $\frac{18}{15}$ by 9.	9. $\frac{51}{50}$ by 3.	14. $\frac{144}{169}$ by 12.
5. $\frac{20}{21}$ by 5.	10. $\frac{57}{50}$ by 3.	15. $\frac{169}{196}$ by 13.

MULTIPLY	MULTIPLY	MULTIPLY
1. $\frac{24}{82}$ by 8.	5. $\frac{41}{51}$ by 17.	9. $\frac{101}{202}$ by 101.
2. $\frac{32}{84}$ by 12.	6. $\frac{78}{96}$ by 16.	10. $\frac{202}{303}$ by 303.
3. $\frac{17}{52}$ by 13.	7. $\frac{92}{78}$ by 13.	11. $\frac{660}{999}$ by 111.
4. $\frac{2}{91}$ by 13.	8. $\frac{45}{54}$ by 18.	12. $\frac{1414}{9998}$ by 14.

MULTIPLY	MULTIPLY	MULTIPLY
1. $21\frac{1}{8}$ by 3.	6. $201\frac{3}{8}$ by 8.	11. $13\frac{1}{8}$ by 13.
2. $4\frac{1}{4}$ by 8.	7. $311\frac{1}{2}$ by 4.	12. $14\frac{8}{28}$ by 14.
3. $5\frac{2}{8}$ by 6.	8. $3\frac{21}{212}$ by 212.	13. $27\frac{27}{86}$ by 18.
4. $9\frac{3}{4}$ by 12.	9. $4\frac{641}{944}$ by 2.	14. $36\frac{41}{91}$ by 13.
5. $21\frac{3}{5}$ by 10.	10. $81\frac{111}{556}$ by 6.	15. $17\frac{21}{84}$ by 12.

CASE II.

149. To multiply an integer by a fraction.

To multiply an integer by a fraction consists in taking a fractional part of the integer: thus, 3 multiplied by $\frac{2}{5}$ is the same as $\frac{2}{5}$ of 3, which is $\frac{6}{5}$, or $1\frac{1}{5}$. Hence,

RULE. — a. *Multiply the integer by the numerator, and write the result over the denominator; or,*

b. *Divide the integer by the denominator, and multiply the quotient by the numerator.*

MULTIPLY (a.)

1. 2 by $\frac{2}{8}$.
2. 5 by $\frac{3}{4}$.
3. 7 by $\frac{4}{5}$.
4. 9 by $\frac{2}{7}$.
5. 11 by $\frac{2}{9}$.
6. 14 by $\frac{3}{8}$.
7. 27 by $\frac{5}{12}$.
8. 41 by $\frac{4}{7}$.
9. 81 by $\frac{3}{11}$.
10. 100 by $\frac{2}{15}$.

MULTIPLY (b.)

11. 6 by $\frac{2}{8}$.
12. 10 by $\frac{4}{5}$.
13. 12 by $\frac{4}{6}$.
14. 21 by $\frac{3}{7}$.
15. 51 by $\frac{2}{17}$.
16. 39 by $\frac{2}{13}$.
17. 91 by $\frac{5}{13}$.
18. 104 by $\frac{3}{26}$.
19. 64 by $\frac{5}{16}$.
20. 58 by $\frac{4}{23}$.

CASE III.

150. To multiply a fraction by a fraction.

Example. — Multiply $\frac{2}{3}$ by $\frac{3}{4}$.

Solution. — $\frac{2}{3} \times 1 = \frac{2}{3}$; $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ of $\frac{2}{3}$, or $\frac{2}{12}$;
 $\frac{2}{3} \times \frac{3}{4} = 3$ times $\frac{1}{4}$ of $\frac{2}{3}$, or $\frac{6}{12}$.

RULE.—*Multiply the numerators of all the several fractions together for a new numerator, and the denominators for a new denominator.*

Suggestion.—The work may be often abbreviated by indicating the multiplication and cancelling the common factors in the numerator and denominator.

MULTIPLY

1. $\frac{7}{8}$ by $\frac{5}{9}$.
2. $\frac{5}{9}$ by $\frac{8}{11}$.
3. $\frac{12}{13}$ by $\frac{13}{17}$.
4. $\frac{9}{16}$ by $\frac{21}{23}$.
5. $\frac{18}{19}$ by $\frac{11}{18}$.
6. $\frac{13}{14}$ by $\frac{3}{11}$.
7. $\frac{21}{23}$ by $\frac{2}{5}$.
8. $\frac{14}{17}$ by $\frac{4}{7}$.
9. $\frac{11}{18}$ by $\frac{5}{8}$.
10. $\frac{101}{304}$ by $\frac{303}{201}$.

MULTIPLY

11. $\frac{12}{13}$ by $\frac{27}{33}$.
12. $\frac{21}{23}$ by $\frac{56}{63}$.
13. $\frac{63}{81}$ by $\frac{13}{21}$.
14. $\frac{42}{78}$ by $\frac{52}{84}$.
15. $\frac{404}{202}$ by $\frac{404}{202}$.
16. $\frac{310}{103}$ by $\frac{103}{310}$.
17. $\frac{14}{98}$ by $\frac{91}{28}$.
18. $\frac{605}{208}$ by $\frac{1012}{1210}$.
19. $\frac{37}{73}$ by $\frac{219}{111}$.
20. $\frac{51112}{21116}$ by $\frac{84460}{204448}$.

CASE IV.

151. To multiply a mixed number by a mixed number.

Example.—Multiply $12\frac{3}{4}$ by $12\frac{3}{4}$.

Solution.— $12 \times 12 = 144$; First partial product.

$12 \times \frac{3}{4} = 9$; Second partial product.

$\frac{3}{4} \times 12 = 9$; Third partial product.

$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$; Fourth partial product.

Hence, $12\frac{3}{4} \times 12\frac{3}{4} = 162\frac{9}{16}$.

MULTIPLY

1. $2\frac{1}{2}$ by $2\frac{1}{2}$.
2. $2\frac{1}{2}$ by $3\frac{1}{2}$.
3. $3\frac{1}{2}$ by $4\frac{1}{2}$.
4. $4\frac{1}{2}$ by $4\frac{1}{2}$.
5. $4\frac{1}{8}$ by $4\frac{1}{8}$.
6. $4\frac{1}{8}$ by $4\frac{1}{8}$.
7. $12\frac{2}{8}$ by $12\frac{2}{8}$.
8. $12\frac{3}{4}$ by $12\frac{3}{4}$.

MULTIPLY

9. $16\frac{3}{4}$ by $16\frac{3}{4}$.
10. $15\frac{3}{4}$ by $15\frac{3}{4}$.
11. $18\frac{3}{4}$ by $18\frac{3}{4}$.
12. $21\frac{4}{8}$ by $23\frac{3}{4}$.
13. $27\frac{2}{7}$ by $17\frac{2}{5}$.
14. $701\frac{1}{8}$ by $107\frac{2}{8}$.
15. $407\frac{2}{8}$ by $704\frac{3}{4}$.
16. $219\frac{1}{8}$ by $921\frac{3}{8}$.

17. A man walks $3\frac{1}{2}$ miles in one hour; how many miles can he walk in $7\frac{1}{8}$ hours?

18. A farm containing $7\frac{1}{8}$ acres cost $\$311\frac{1}{4}$; how much is a similar farm containing $4\frac{1}{2}$ times as many acres worth?

19. A vessel sailed $13\frac{1}{8}$ miles in one hour; at the same rate, how many miles would it sail in $13\frac{1}{8}$ hours?

20. At $16\frac{3}{4}$ cents a pound, what will $16\frac{3}{4}$ pounds of butter cost?

DIVISION OF FRACTIONS.

152. Division of fractions consists in finding how many times, or parts of a time, one fraction is contained in another.

- a. A fraction may be divided by a whole number.
- b. A whole number may be divided by a fraction.
- c. A fraction may be divided by a fraction.

CASE I.

153. To divide a fraction by an integer.

RULE. — a. *Divide the numerator of the fraction by the integer.* (Principle 3.)

b. *Multiply the denominator by the integer.* (Principle 2.)

- | | |
|----------------------------------|----------------------------------|
| 1. Divide $\frac{4}{5}$ by 2. | 6. Divide $\frac{80}{91}$ by 9. |
| 2. Divide $\frac{10}{19}$ by 5. | 7. Divide $\frac{90}{78}$ by 13. |
| 3. Divide $\frac{21}{41}$ by 7. | 8. Divide $\frac{78}{90}$ by 3. |
| 4. Divide $\frac{51}{52}$ by 17. | 9. Divide $\frac{25}{41}$ by 4. |
| 5. Divide $\frac{88}{58}$ by 19. | 10. Divide $\frac{18}{19}$ by 7. |

CASE II.

154. To divide an integer by a fraction.

Example. — Divide 9 by $\frac{2}{3}$.

Solution. — $9 \div 1 = 9$; $9 \div \frac{1}{3} = 3 \times 9$, or 27; then $9 \div \frac{2}{3} = \frac{1}{2}$ of 3×9 , or $\frac{3 \times 9}{2} = 13\frac{1}{2}$. Ans.

RULE. — a. *Multiply the integer by the denominator of the fraction, and divide the product by the numerator; or, —*

b. *Divide the integer by the numerator of the fraction, and multiply the quotient by the denominator.*

- | | |
|------------------------------------|--|
| 1. Divide 4 by $\frac{2}{8}$. | 11. Divide 51 by $\frac{17}{20}$. |
| 2. Divide 8 by $\frac{4}{5}$. | 12. Divide 57 by $\frac{19}{20}$. |
| 3. Divide 10 by $\frac{5}{7}$. | 13. Divide 68 by $\frac{17}{25}$. |
| 4. Divide 12 by $\frac{6}{11}$. | 14. Divide 45 by $\frac{15}{16}$. |
| 5. Divide 14 by $\frac{7}{8}$. | 15. Divide 28 by $\frac{14}{15}$. |
| 6. Divide 15 by $\frac{8}{8}$. | 16. Divide 91 by $\frac{13}{16}$. |
| 7. Divide 6 by $\frac{8}{9}$. | 17. Divide 104 by $\frac{52}{58}$. |
| 8. Divide 20 by $\frac{5}{12}$. | 18. Divide 101 by $\frac{101}{22}$. |
| 9. Divide 21 by $\frac{7}{10}$. | 19. Divide 1111 by $\frac{101}{888}$. |
| 10. Divide 21 by $\frac{11}{22}$. | 20. Divide 222 by $\frac{111}{112}$. |

CASE III.

155. To divide a fraction by a fraction.

Example. — Divide $\frac{3}{4}$ by $\frac{5}{8}$.

Solution. — $\frac{3}{4} \div 1 = \frac{3}{4}$; $\frac{3}{4} \div \frac{1}{8} = \frac{6 \times 3}{4}$, or $1\frac{3}{2}$;

$\frac{3}{4} \div \frac{5}{8} = \frac{1}{5}$ of $\frac{6 \times 3}{4} = \frac{6 \times 3}{5 \times 4}$, or $\frac{18}{20}$, or $\frac{9}{10}$. Ans.

RULE. — *Invert the divisor and proceed as in multiplication of fractions.*

- | | |
|---|--|
| 1. Divide $\frac{3}{4}$ by $\frac{3}{4}$. | 9. Divide $\frac{1}{2}$ by $\frac{1}{8}$. |
| 2. Divide $\frac{8}{4}$ by $\frac{2}{8}$. | 10. Divide $2\frac{1}{2}$ by $2\frac{1}{8}$. |
| 3. Divide $\frac{4}{5}$ by $\frac{5}{8}$. | 11. Divide $4\frac{1}{5}$ by $5\frac{1}{4}$. |
| 4. Divide $\frac{5}{6}$ by $\frac{4}{5}$. | 12. Divide $5\frac{1}{6}$ by $4\frac{1}{4}$. |
| 5. Divide $\frac{5}{16}$ by $\frac{5}{4}$. | 13. Divide $7\frac{1}{12}$ by $7\frac{1}{7}$. |
| 6. Divide $\frac{4}{5}$ by $\frac{6}{8}$. | 14. Divide $9\frac{1}{18}$ by $8\frac{1}{9}$. |
| 7. Divide $\frac{5}{4}$ by $\frac{6}{5}$. | 15. Divide $10\frac{8}{11}$ by $1\frac{2}{88}$. |
| 8. Divide $1\frac{1}{20}$ by $2\frac{2}{8}$. | 16. Divide $\frac{49}{54}$ by $1\frac{4}{18}$. |

DIVIDE

17. $\frac{1}{8}$ by $\frac{1}{2}$.

18. $\frac{21}{81}$ by $\frac{7}{41}$.

19. $\frac{42}{9}$ by $\frac{8}{81}$.

20. $21\frac{1}{8}$ by $6\frac{1}{21}$.

21. $17\frac{7}{9}$ by $9\frac{7}{17}$.

22. $\frac{1010}{808}$ by $\frac{10}{80}$.

23. $210\frac{1}{4}$ by $120\frac{1}{5}$.

24. $401\frac{1}{8}$ by $104\frac{2}{3}$.

DIVIDE

25. $291\frac{1}{2}$ by $192\frac{1}{2}$.

26. $741\frac{1}{5}$ by $147\frac{1}{5}$.

27. $924\frac{6}{7}$ by $\frac{1001}{14}$.

28. $81\frac{3}{8}$ by $91\frac{3}{8}$.

29. $91\frac{3}{8}$ by $81\frac{3}{8}$.

30. $(21\frac{2}{3} \div 6\frac{1}{2})$ by $(6\frac{2}{3} \div 21\frac{1}{2})$.

31. $298\frac{2}{5}$ by $297\frac{4}{5}$.

32. $425\frac{3}{4}$ by $426\frac{1}{4}$.

156. To find what part one fraction is of another.

Example. — $\frac{1}{2}$ is what part of $\frac{1}{8}$?

Solution. — *a.* $\frac{1}{2} = \frac{4}{8}$; $\frac{1}{8} = \frac{1}{8}$;

b. $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{2}{8}$, and $\frac{4}{8}$ is $\frac{2}{1}$ of $\frac{2}{8}$.

Therefore, $\frac{1}{2}$ is $\frac{2}{1}$ of $\frac{1}{8}$.

RULE. — *Reduce the fractions to be compared to equivalent fractions having a common denominator, and compare the numerators.*

1. $\frac{1}{4}$ is what part of $\frac{1}{2}$?
2. $\frac{1}{2}$ is what part of $\frac{2}{3}$?
3. $\frac{2}{3}$ is what part of $\frac{3}{4}$?
4. $\frac{3}{4}$ is what part of $\frac{2}{3}$?
5. $\frac{4}{5}$ is what part of $\frac{8}{9}$?
6. $\frac{8}{9}$ is what part of $\frac{4}{5}$?
7. $1\frac{1}{8}$ is what part of $2\frac{1}{2}$?
8. $2\frac{1}{2}$ is what part of $3\frac{1}{2}$?
9. $4\frac{1}{5}$ is what part of $12\frac{3}{5}$?
10. $\frac{2}{3}$ of $\frac{3}{4}$ is what part of $\frac{1}{2}$ of $\frac{11}{12}$?
11. $18\frac{3}{4}$ is what part of $3\frac{4}{18}$?

12. $1501\frac{1}{4}$ is what part of $3002\frac{1}{8}$?
13. $\frac{\frac{2}{8} \text{ of } \frac{11}{12}}{\frac{1}{81} \text{ of } 5\frac{1}{2}}$ is what part of $\frac{\frac{2}{8} \text{ of } 3\frac{3}{8}}{\frac{1}{8} \text{ of } 5\frac{1}{2}}$?

GENERAL REVIEW.

1. What fraction added to $\frac{2}{8}$ will make $\frac{3}{4}$?
2. What fraction taken from $\frac{4}{5}$ will make $\frac{5}{8}$?
3. What fraction multiplied by $\frac{7}{8}$ gives $\frac{5}{8}$?
4. What fraction divided by $\frac{5}{12}$ gives $\frac{21}{80}$?
5. $\frac{2}{7}$ is what part of 2?
6. $\frac{2}{5}$ is what part of 3?
7. 3 is $\frac{1}{2}$ of what number?
8. $\frac{1}{2}$ of 6 is $\frac{1}{8}$ of what number?
9. $\frac{3}{4}$ of $\frac{2}{9}$ of 36 is $\frac{2}{8}$ of what number?
10. 3 times $\frac{2}{5}$ of 20 is how many times 2?
11. $\frac{1}{8}$ of 27 is what part of $\frac{2}{5}$ of 30?
12. $\frac{1}{2}$ of 12 is $\frac{2}{8}$ of what part of 20?
13. 40 is $\frac{2}{8}$ of what part of 100?
14. If 4 hats cost \$8, what will 1 hat cost?
15. If 10 horses cost \$1000, what will 1 horse cost?
16. Find the cost of 1 watch when 4 watches cost \$500.
17. Find the cost of 1 cord of wood when 7 cords cost \$56.
18. What will 8 pounds of butter cost if 2 pounds cost 25 cts.?
19. If one man do a piece of work in 4 days, what part of the work can he do in 1 day?

20. If one man can do a piece of work in 12 days, what part of the work can he do in 4 days?

21. If two men can do a piece of work in 8 days, what part of the work can they do in 2 days?

22. If A can do a piece of work in 3 days, and B can do it in 4 days, what part of the work can each man do in 1 day? What part of it can they both do in 1 day? What part can both do in 2 days?

23. George can do some work in 5 days, John can do the same work in 3 days, and James can do it in 10 days; what part of the work can each do in 1 day? What part of it can George and John do in 1 day? What part can George and James do in 1 day? What part can John and James do in 1 day? What part of the work can all three do in 1 day? In 2 days?

24. A and B can do a piece of work in 9 days, and B alone can do it in 20 days; what part of it can both do in 1 day? What part can B alone do in 1 day? What part can A do alone in 1 day? How long would it take A alone to do the whole work?

25. A and B can do a piece of work in 12 days; what part of it can they do in 1 day?

26. A and C can do the same work in 15 days; what part of it can they do in 1 day?

27. B and C can do the same piece of work in 20 days; what part of it can they do in 1 day?

28. If a man can do $\frac{1}{10}$ of a piece of work in 1 day, how long will it take him to do the whole work?

29. If a man can do $\frac{2}{12}$ of a piece of work in one day, how long will it take him to do the whole work?

30. If 4 men can do a piece of work in 20 days, in how many days can 8 men do it?

31. If 4 men can do a piece of work in 12 days, by working 2 hours each day, in how many days can 14 men do the same work, working 1 hour each day?

32. What will 40 tons of coal cost at $\$8.37\frac{1}{2}$ per ton?

Ans. $\$335$.

33. If a man travels $\frac{8}{11}$ of a mile in 1 hour, how far would he travel in $\frac{1}{9}\frac{1}{2}$ of an hour?

34. How many pounds of butter can be bought for $\$1.24\frac{1}{2}$, at $41\frac{1}{2}$ cents per pound?

35. At $62\frac{1}{2}$ cents per pound, how many pounds of tea can be bought for $\$5.62\frac{1}{2}$?

36. How much more will 7 barrels of flour cost at $\$5\frac{1}{2}$ per barrel than 105 bushels of potatoes at $27\frac{1}{4}$ cents per bushel?

37. What number increased by $\frac{2}{8}$ of $\frac{3}{4}$ of $9\frac{1}{2}$ will make 5?

38. $12\frac{1}{2}$ times a certain number equals $48\frac{3}{4}$; what is the number?

39. Joseph has $\frac{5}{8}$ of $\frac{3}{8}$ of $\$32$, which is 3 times $\frac{2}{5}$ of the money William has; how much has each?

40. 27 is what part of $\frac{3}{11}$ of $16\frac{1}{2}$?

41. A man bought 9 cords of wood at $\$4.33\frac{1}{3}$ a cord, and paid for it in labor at $\$1.50$ a day. How many days did he work?

42. Coal oil at 15 cents a gallon was exchanged for 20 pounds of tea at $75\frac{1}{2}$ cents per pound, and 37 pounds of butter at $12\frac{1}{2}$ cents per pound, how many gallons of coal oil were required?

43. A miller owned $\frac{2}{7}$ of a mill and sold $\frac{2}{3}$ of his share; what part of the mill did he still own?

44. A and B own a farm. A owns $\frac{2}{7}$ of it, and B the remainder. B sells $\frac{2}{3}$ of his share to A; what part of the farm does each own?

45. A man sold a watch for \$62.50, which was $\frac{5}{7}$ of the cost of a new one. Find the difference in the price of the two watches.

46. Reduce $\frac{21}{50}$ to an equivalent fraction having 7 for its denominator.

47. A can do twice as much work as B; how many times B's work can both do?

48. B can do 5 times as much work as C; how many times C's work can both do?

49. If A can do $2\frac{1}{2}$ times as much work as B, what part of A's work can B do?

50. A and B together do a piece of work in 12 days; what part of the work do both do in 1 day? If A does 3 times as much as B, how many times B's work do they both do? What part of the whole work does B do in one day? What part of the whole work does A do in one day? How long will it take B to do the work alone? How long will it take A to do the work alone?

51. Two men do a piece of work in 18 days. If the first man works 3 times as fast as the second, in how many days can each do the work alone?

52. How much greater, or less, is $\frac{8}{15}$ of $\frac{44}{61}$ than $\frac{6}{18}$ of $\frac{48}{62}$?

53. Reduce $(2\frac{1}{2} \times 5\frac{1}{3}) \div (4\frac{1}{5} \times 5\frac{1}{4})$ to simplest form.

54. How much will 17 umbrellas cost, if 6 cost \$14.37 $\frac{1}{2}$?

55. $\frac{2}{3}$ of a pound of indigo cost 12 $\frac{1}{2}$ cents, what must be paid for 1 $\frac{3}{8}$ pounds at the same rate?

56. What is the value of 3 bales of cotton, each weighing 440 $\frac{1}{2}$ pounds, when 1 pound sells for 13 $\frac{1}{2}$ cents?

57. 720 bushels of oats are worth 120 bushels of wheat. What is the value of the oats, if wheat is worth 90 cents a bushel?

58. 37 $\frac{1}{2}$ yards of brussels carpet are worth 225 bushels of potatoes at 25 cents a bushel. What is the price per yard?

59. A father left a fortune to his three children. The oldest was to have $\frac{2}{5}$ of it, the second was to have $\frac{2}{11}$ of it, and the youngest was to have the rest. By this arrangement the oldest received \$10000 more than the youngest. Find the value of the estate, and the amount each received.

60. Add $\frac{1}{8}$ of $\frac{4}{5}$ of $\frac{7}{9}$ and $\frac{2}{9}$ of $\frac{5}{11}$ of 7 $\frac{1}{8}$, and divide

$$\frac{1}{34\frac{1}{8}}$$
the sum by $\frac{2}{3}$ of $\frac{5}{2}$ of 9.

61. Twice $\frac{5}{14}$ of $\frac{6}{18}$ of 91 is $\frac{2}{10}$ of how many times $\frac{2}{5}$ of six times $\frac{2}{5}$ of 17 $\frac{1}{2}$?

DECIMAL FRACTIONS.

157. Decimal fractions are fractions whose denominator, usually understood, is ten, or some power of ten.

158. A decimal fraction always expresses a certain number of tenths, hundredths, thousandths, etc., of a unit.

Thus : .5, which is read five-tenths.

.05, which is read five-hundredths.

.005, which is read five-thousandths.

NOTATION OF DECIMALS.

159. The decimal point (.) determines the **denomination** of the fraction.

160. The denominator of a common fraction may be any number whatever, and is always expressed.

The scale in decimal fractions is the same as that in whole numbers.

This is shown below in writing the following numbers :—

20000.		Twenty thousand.
2000.		Two thousand.
200.		Two hundred.
20.		Twenty.
2.		Two units.
.2		Two tenths.
.02		Two hundredths.
.002		Two thousandths.
.0002		Two ten-thousandths.
.00002		Two hundred-thousandths.

161. Thus, it is shown, —

1st. That the local value of a digit is only *one-tenth* as great in the *fourth* as it is in the *fifth* place at the left of the point.

2d. That the local value of a digit is only one-tenth as great in the *third* as in the *fourth* place at the left of the point.

3d. That the local value of a digit is only one-tenth as great in the *second* as it is in the *third* place at the left of the point.

4th. That the local value of a digit is only one-tenth as great in the *first* as it is in the *second* place at the left of the point.

5th. That the local value of a digit is only one-tenth as great in the *first* place at the right of the point as it is in the *first* place at the left of the point.

GENERAL LAW.

Finally. While numbers increase in a ten-fold ratio from the point toward the left, they decrease in a ten-fold ratio from the point toward the right.

WRITING DECIMALS.

RULE. — *Write as in whole numbers, and place the decimal point so as to express the denomination required.*

162. In decimals, as in whole numbers, each digit used in expressing a number may have a simple or a local value.

163. The local value of any digit depends upon its position with reference to the decimal point.

READING DECIMALS.

RULE. — *Read as in whole numbers, giving the name of the lowest denomination in the decimal read.*

NOTATION AND NUMERATION OF DECIMALS.

How many places are required —

1. To express thirty-three thousand?
2. To express thirty-three hundred?
3. To express three hundred and thirty-three units?
4. To express thirty-three units?
5. To express thirty-three tenths?
6. To express three hundred and thirty-three *tenths*?
7. To express thirty-three hundredths?
8. To express three hundred and thirty-three *hundredths*?
9. To express thirty-three thousandths?
10. To express three hundred and thirty-three *thousandths*?
11. To express thirty-three ten-thousandths?
12. To express three hundred and thirty-three *ten-thousandths*?
13. To express three *tenths*?
14. To express three units and three *tenths*?

Analyze and write —

1. Thirty-three thousand.
2. Thirty-three hundred.
3. Three hundred and thirty-three units.
4. Thirty-three units.
5. Thirty-three tenths.
6. Three hundred and thirty-three *tenths*.
7. Thirty-three hundredths.
8. Three hundred and thirty-three *hundredths*.

- 166.** The value of a common fraction may be expressed decimally, thus :

167. The value of a decimal may be expressed in the form of a common fraction, thus :

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ADDITION.

170. Addition of decimals consists in **finding the sum** of two or more decimals.

RULE. — *Write the decimals to be added so that units of the same order shall stand in the same column, and add as in simple numbers.*

1. Write, and add three-tenths, seven-tenths, four-tenths, nine-tenths, eight tenths.

2. Write, and add twelve-hundredths, thirteen-hundredths, fourteen-hundredths, fifteen-hundredths.

3. Write, and add twenty-one thousandths, three hundred and twenty-one thousandths, five hundred and seventeen thousandths.

4. Write, and add twenty-seven thousand and eight ten-thousandths, thirty-seven thousand and one ten-thousandths, forty-seven thousand eight hundred and forty-one thousandths.

ADD	ADD
5. 23124.7682	6. 7865432.12345678
231.247682	123456.782345687
23.1247682	786.543212345678
2312.47682	87654.3212345678
.231247682	8.7654321234578
.00231247682	23.6894

Add

7. Twenty-seven hundred and twenty-seven thousandths, twenty-seven hundred and twenty-seven hun-

dredths, twenty-seven hundred and twenty-seven thousandths, twenty-seven hundred and twenty-seven tenths, twenty-seven hundred and twenty-seven hundred-thousandths.

Add

8. Forty-one hundred and forty-one tenths, thirty-one hundred and thirty-one hundredths, twenty-one hundred and twenty-one thousandths, ten hundred and ten ten-thousandths, fifty-one hundred and fifty-one hundred-thousandths.

Add

9. Twenty-five thousand and twenty-five tenths,
 Twenty-five hundred and twenty-five hundredths,
 Twenty-five thousandths,
 Twenty-five hundred-thousandths,
 Twenty-five millionths,
 Twenty-five thousand, and twenty-five tenths,
 Twenty-five hundred, and twenty-five hundredths,
 Twenty-five, and twenty-five tenths.

SUBTRACTION.

171. Subtraction of decimals consists in finding the difference between two decimals.

RULE. — *Write the subtrahend under the minuend, so that the same orders shall stand in the same column, and subtract as in simple numbers.*

1. From eight-tenths take seven-tenths.
2. From five hundredths take three hundredths.

3. From four thousandths take one thousandth.

4. From nine ten-thousandths take three ten-thousandths.

5. From six hundred-thousandths take six millionths.

6. From 237.84562

Take 51.2164

7. From 8.7642

Take .94328

8. From 1.0783147

Take .9874156

9. From .07256139

Take .00002799

10. From twenty-nine thousandths
Take twenty-nine ten-thousandths.

11. From seventy-two millionths
Take seventy-two hundred-millionths.

12. From seventy-two millionths
Take seventy-two ten-millionths.

13. From seventy-seven millionths plus twenty-one ten-millionths
Take sixty-six millionths plus twenty-two ten-millionths.

14. From six hundred and five hundred-millionths
Take five hundred and six hundred-millionths.

15. Find the difference between one hundred-millionth and one billionth.

16. What number added to nineteen hundred-thousandths will make nineteen thousandths?

17. The sum of three numbers is ninety-two ten-millionths; the first is seven ten-millionths, and the second is seven hundred-millionths; what is the third?

MULTIPLICATION.

172. Multiplication of decimals consists in finding the product of two decimals.

173. Decimals are multiplied as simple numbers, regard being had to the decimal point.

174. The position of the decimal point in the product depends upon the following principles :

FIRST PRINCIPLE.

Dividing the multiplicand by 10, 100, 1000, etc., divides the product by 10, 100, 1000, etc.

SECOND PRINCIPLE.

Dividing the multiplier by 10, 100, 1000, etc., divides the product by 10, 100, 1000, etc.

THIRD PRINCIPLE.

Dividing both multiplier and multiplicand by 10, 100, 1000, etc., divides the product by 100, 10000, 1000000, etc.

Illustration. — Multiply 2.56 by 25.6.

Solution. — Regarding the multiplier and multiplicand as integers, the product is 65536; but since the multiplier, regarded as an integer, is 10 times too large, and since the multiplicand, regarded as an integer, is 100 times too large, the product must be 10 times 100, or 1000 times too large; by Third Principle, the product thus obtained must therefore be divided by 1000, which consists in pointing off three decimal places, the number of decimal places in both factors. Hence the

$$\begin{array}{r}
 2.56 \\
 25.6 \\
 \hline
 1536 \\
 1280 \\
 512 \\
 \hline
 65.536
 \end{array}$$

RULE. — *Multiply as in simple numbers, and point off in the product as many decimal places as there are in both multiplicand and multiplier.*

If the product of the significant figures in both factors does not give the number of decimal places required, prefix ciphers to supply the deficiency.

MULTIPLY

1. .5 by .5
2. .5 by .35
3. .35 by .35
4. .334 by .4
5. .334 by .04
6. .334 by .334
7. .888 by .2
8. .888 by .22
9. 8888 by .222
10. 8888 by .2222

MULTIPLY

11. .333 by .33
12. .4444 by .03
13. .53535 by .023
14. 657843 by .673
15. .121212 by .23232
16. .373737 by .262626
17. 34.34 by 24.24
18. 19.91 by 91.19
19. 87.789 by 78.987
20. 999.99 by 12.3456

21. Find the product of $3.4 \times 12.5 \times 10 \times .3125 \times .08 \times 1$

22. Find the product of $.4 \times 2 \times .5 \times 2.5 \times .2 \times 5 \times 1.28 \times .78125$

23. Multiply eighty-five hundred-millionths by seven hundred and forty-five ten-millionths.

24. Multiply six hundred and sixty-six thousand five hundred and fifty-five billionths by five hundred and fifty-five thousand six hundred and sixty-six millionths ; and multiply this product by ninety-nine hundred and ninety-nine tenths.

DIVISION OF FRACTIONS.

175. Division of decimals consists in finding how many times one decimal is contained in another.

176. Division of decimals is performed as in simple numbers, regard being had to the decimal point.

177. The position of the decimal point in the quotient depends upon the following principles :

FIRST PRINCIPLE.

Dividing the divisor by 10, 100, 1000, etc., multiplies the quotient by 10, 100, 1000, etc.

SECOND PRINCIPLE.

Dividing the dividend by 10, 100, 1000, etc., divides the quotient by 10, 100, 1000, etc.

Illustration. — Divide .256 by 25.6

Solution. — Regarding both dividend and divisor as integers, the quotient is 1; but since the dividend, regarded as an integer, is 1000 times too large, the quotient is 1000 times too large, and should be divided by 1000, which gives .001. But since the divisor, regarded as an integer, is 10 times too large, this quotient is ten times too small, and should be $10 \times .001$, or .01, which is the number of decimal places in the dividend in excess of those in the divisor. Hence the

RULE. — *Divide as in simple numbers, and point off in the quotient as many decimal places as those in the dividend exceed those in the divisor.*

Suggestion. — a. *When the divisor contains a greater number of decimal places than the dividend, make the places in the dividend equal to those in the divisor by annexing ciphers to the dividend.*

b. *When the decimal places in divisor and dividend are equal the quotient is a whole number.*

c. *Each cipher annexed to the dividend, in continuing the division, increases the number of decimal places in the quotient.*

DIVIDE

1. 625 by 625
2. 625 by 62.5
3. 625 by 6.25
4. 625 by .625
5. 62.5 by .625
6. 6.25 by .625
7. .625 by .625
8. 1.25 by 12.5
9. .032 by .016
10. .016 by .32

DIVIDE

11. 17.28 by .12
12. 1.728 by 1.2
13. .1728 by .012
14. .0064 by .00000032
15. .00000032 by .0064
16. .2222 by .00000011
17. 32.75 by 4.25
18. 4.756 by .375
19. 101.202 by 202.101
20. 245.7 by .0000125

21. The product of two numbers is eight hundred and twenty-five ten-thousandths, and one of them is one hundred and twenty-five millionths; what is the other number?

22. If 27 horses cost \$1684.125, what did one cost?

23. Multiply twelve hundred-thousandths by four-teen ten-thousandths, and divide the result by the product of forty-two ten-millionths by fifty-four tenths.

REDUCTION OF DECIMALS.

178. Reduction of decimals consists in changing

a. A decimal to the form of a common fraction.

b. A common fraction to the form of a decimal.

c. A decimal of one denomination to a decimal of different denomination.

CASE I.

179. To reduce a decimal to the form of a common fraction.

Example. — Change .0825 to an equivalent common fraction.

Solution. — Since the given decimal contains four places it expresses ten-thousandths. Writing the significant figures as the numerator and 10000 as the denominator, we get $\frac{825}{10000}$, which, reduced to its lowest terms, gives $\frac{33}{400}$. Hence the

RULE. — *As the numerator write the given decimal, omitting the decimal point and the ciphers prefixed to the decimal part; as the denominator write 1 with as many ciphers annexed as there are places in the given decimal.*

Reduce to common fractions and to lowest terms :

1. .5	11. .875	21. 33.22
2. .25	12. .9375	22. 22.35
3. .625	13. .048	23. 7.28
4. .05	14. .0625	24. 44.444
5. .025	15. 35.75	25. 40.044
6. .0625	16. 47.25	26. 11.2244
7. .012	17. 54.725	27. 224.411
8. 92.5	18. 811.125	28. 81.1818
9. 6.25	19. 11.85	29. 18.8181
10. 62.5	20. 111.222	30. 19.911

CASE II.

180. To reduce a common fraction to a decimal.

Example. — Change $\frac{1}{8}$ to an equivalent decimal.

Solution.—This fraction expresses an unexecuted division which is to be performed. Since 8 is not contained in 1 unit, we reduce the unit to tenths, which gives 10 tenths; $\frac{1}{8}$ of 10 tenths is .1, with a remainder of .2, or .20. $\frac{1}{8}$ of .20 is .02, with a remainder of .04, or .040. $\frac{1}{8}$ of .040 is .005. The quotient, then, is .125; therefore, $\frac{1}{8} = .125$. Hence the

RULE.—*Divide the numerator by the denominator, annexing ciphers to the numerator, and point off as in division of decimals.*

Change the following to equivalent decimals :

1. $\frac{1}{2}$.	8. $\frac{21}{84}$.	15. $\frac{1}{8}$.
2. $\frac{1}{4}$.	9. $\frac{24}{25}$.	16. $\frac{2}{8}$.
3. $\frac{1}{5}$.	10. $\frac{7}{25}$.	17. $\frac{1}{8}$.
4. $\frac{3}{8}$.	11. $\frac{3}{18}$.	18. $\frac{5}{6}$.
5. $\frac{3}{4}$.	12. $\frac{29}{125}$.	19. $\frac{1}{7}$.
6. $\frac{9}{18}$.	13. $\frac{15}{125}$.	20. $\frac{5}{7}$.
7. $\frac{15}{82}$.	14. $\frac{4}{12}$.	21. $\frac{9}{14}$.

181. The value of a common fraction can be exactly expressed decimally only when the prime factors of its denominator are factors of 10 ; as 2 and 5.

182. The number of decimal places required to express a common fraction decimally is determined by the greatest number of times 2 or 5 occurs as a factor of its denominator.

Have the following fractions exact decimal equivalents? and how many places are required to express them decimally?

Illustration.—Has $\frac{3}{4}$ an exact decimal equivalent?

Solution.—The denominator $4 = 2 \times 2$, and since 2 is a factor of 10, the value of $\frac{3}{4}$ may be exactly expressed decimally. The

number of places required to express it is two, since 2 occurs twice as a factor of the denominator. Hence, $\frac{3}{4} = .75$.

2. $\frac{1}{2}$.	7. $\frac{1}{8}$.	12. $\frac{1}{32}$.
3. $\frac{1}{4}$.	8. $\frac{3}{8}$.	13. $\frac{9}{32}$.
4. $\frac{1}{5}$.	9. $\frac{5}{8}$.	14. $\frac{1}{3}$.
5. $\frac{2}{5}$.	10. $\frac{1}{16}$.	15. $\frac{2}{3}$.
6. $\frac{9}{25}$.	11. $\frac{5}{16}$.	16. $\frac{5}{6}$.

183. When a common fraction has no exact decimal equivalent the resulting decimal is called a **circulating decimal**. Thus,

$$\begin{aligned}\frac{1}{3} &= .3\frac{1}{3}, \text{ or } .33\frac{1}{3}, \text{ or } .333\frac{1}{3}, \text{ or } .333333\frac{1}{3} \\ \frac{2}{6} &= .6\frac{2}{3}, \text{ or } .66\frac{2}{3}, \text{ or } .666\frac{2}{3}, \text{ or } .666666\frac{2}{3} \\ \frac{1}{6} &= .16666666\frac{2}{3}\end{aligned}$$

NOTE.—The subject of circulating decimals belongs properly to higher arithmetic.

CASE III. in the reduction of decimals belongs to the subject of compound denominate numbers. (See Art. 255.)

REVIEW.

1. Add 256.764, 2.75683, .00000001, 20.20202, 1.000126, 3.00001

2. From .00004673 take .0000279341

3. Multiply 1.0001, 2.0002, 3.0003, 4.0004, 5.0005, 6.0006, .0007

4. Divide 93.75 by 3265096.575 to four significant figures in the quotient.

5. How many casks, each containing 10.485 gallons, can be filled from 83.88 gallons?

6. A pile of cord-wood is 40.16 feet long, 7.04 feet

high. How much is it worth at \$4.75 a cord, there being 128 cubic feet in one cord?

7. How many loads in the above pile of wood, if each load contains 1.25 cords?

8. Find the value of $(7.2372 \times 1.001002) \div (2.002001 \times 1.30201)$.

COMPOUND NUMBERS.

184. A compound number is a concrete number consisting of different denominations.

185. Changing compound numbers from one denomination to another, without changing the value expressed, is called *Reduction*.

186. *Reduction descending* consists in changing from a higher to a lower denomination, and is performed by multiplication.

187. *Reduction ascending* consists in changing from a lower to a higher denomination, and is performed by division.

188. Compound numbers are used in expressing the different Currencies, Weights, and Measures.

UNITED STATES CURRENCY.

189. UNITED STATES CURRENCY is sometimes called FEDERAL MONEY, and is the legal currency of the UNITED STATES.

190. This currency is based upon the decimal scale ; and all operations in this currency are similar to those in whole numbers and decimal fractions. (Art. 83.)

191. The operations in compound numbers are based on varying scales.

192. DIFFERENT SCALES.

1. In United States currency the scale is 10.
 2. In English Money the scale is 4, 12, 20.
 3. In Long Measure the scale is 12, 3, $5\frac{1}{2}$, 40, 8.
 4. In Square Measure the scale is 144, 9, $30\frac{1}{4}$, 40, 4, 640.
 5. In Cubic Measure the scale is 1728, 27, 8, 128.
 6. In Dry Measure the scale is 2, 8, 4.
 7. In Liquid Measure the scale is 4, 2, 4, $31\frac{1}{2}$.
 8. In Apothecaries' Weight the scale is 20, 3, 8, 12.
 9. In Troy Weight the scale is 24, 20, 12.
 10. In Avoirdupois Weight the scale is 16, 25, 4, 20.
- Other weights and measures have other scales.

ENGLISH MONEY.

193. ENGLISH MONEY is the legal currency of GREAT BRITAIN. The denominations are *Farthing*, *Penny*, *Shilling*, *Pound*.

TABLE OF ENGLISH MONEY.

4 Farthings (far.).....	make 1 Penny.	d.
12 Pence.....	1 Shilling.	s.
20 Shillings.....	1 Pound	£.
	or	
	Sovereign.	sov.

Example 1. — Reduce 2£ 4s. 5d. 3 far. to farthings.

Solution. — 2£ 4s. 5d. 3 far.

20 No. of shillings in 1£.

40 No. of shillings in 2£.

4 Add 4 shillings.

44 No. of shillings in 2£ 4s.

12 No. of pence in a shilling.

528 No. of pence in 44s.

5 Add 5 pence.

533 No. of pence in 2£ 4s. 5d.

4 No. of farthings in a penny.

2132 No. of farthings in 533 pence.

3 Add 3 farthings.

2135 No. of farthings in 2£ 4s. 5d. 3 far.

Example 2. — Reduce 2135 farthings to higher denominations.

Solution. — No. of farthings in a penny	4)2135 far.
No. of pence in a shilling	12)533 + 3 far.
No. of shillings in a pound	20)44 + 5d.
	2£ + 4s.

Hence, in 2135 farthings there are . . . 2£ 4s. 5d. 3 far.

3. Reduce 5£ 3s. 7d. 2 far. to farthings.
4. Reduce 7£ 2s. to farthings.
5. Reduce 821 farthings to higher denominations.
6. Reduce 9012 farthings to higher denominations.

WEIGHTS AND MEASURES.

194. Some legal standard, or unit, of measure is used in estimating *extension*, *capacity*, *weight*, etc.

a. In Extension the *yard* is the unit of measure, which is subdivided into *feet* and *inches*.

b. In Capacity the unit varies according to the kind of capacity under consideration.

1. In *Dry Measure* the unit is the *bushel*, which contains 2150.4 *cubic inches*.

2. In *Liquid Measure* the unit is the *gallon*, which contains in *Wine Measure* 231 *cubic inches*, and in *Beer Measure* 282 *cubic inches*.

c. In *Weights* the *Troy Pound*, which contains 5760 *grains*, is the *unit*.

d. In measuring *Time*, the *mean solar day* is the *unit*.

e. In *Angular Measure* one *degree* is the *unit*.

195. The most important measures of extension are,

1. **Linear Measure.**

2. **Square Measure.**

3. **Cubic Measure.**

196. **LINEAR or LONG MEASURE** is used in measuring distances.

LONG MEASURE.

12 Inches (in.).....	make 1 Foot.	<i>ft.</i>
3 Feet.....	1 Yard.	<i>yd.</i>
5½ Yards.....	1 Rod.	<i>rd.</i>
40 Rods.....	1 Furlong.	<i>fur.</i>
8 Furlongs.....	1 Mile.	<i>mi.</i>
5280 Feet.....	1 Mile.	

SURVEYORS' MEASURE.

7.92 Inches.....	make 1 Link.	<i>l.</i>
25 Links.....	1 Rod.	<i>rd.</i>
4 Rods.....	1 Chain	<i>ch.</i>
80 Chains.....	1 Mile.	<i>mi.</i>

Example 1. — Reduce 2 *mi.* 2 *fur.* 12 *rd.* 2 *yds.* 1 *ft.* 5 *in.* to inches.

Solution. — 2 *mi.* 2 *fur.* 12 *rd.* 2 *yds.* 1 *ft.* 5 *in.*

8	No. of furlongs in a mile.
16	No. furlongs in 2 miles.
2	
18	No. furlongs in 2 miles, 2 furlongs.
40	No. rods in a furlong.
720	No. rods in 18 furlongs.
12	
732	No. rods in 18 furlongs, 12 rods.
5½	No. yards in a rod.
3660	
366	
4026	No. yards in 732 rods.
2	
4028	No. yards in 732 rods, 2 yards.
3	No. feet in a yard.
12084	No. feet in 4028 yards.
1	
12085	No. feet in 4028 yards, 1 foot.
12	No. inches in a foot.
145020	No. inches in 12085 feet.
5	
145025	No. inches in 12085 feet, 5 inches.

Hence, 2 *mi.* 2 *fur.* 12 *rd.* 2 *yds.* 1 *ft.* 5 *in.* = 145025 inches.

Example 2. — Reduce 145025 inches to higher denominations.

Solution.—No. inches in a foot, $12 \overline{)145025}$ in.

No. feet in a yard, $3 \overline{)12085}$ ft. + 5 in.

No. of yards in a rod, $5\frac{1}{2} \overline{)4028}$ yds. + 1 ft.
 $\begin{array}{r} 2 \\ \underline{2} \end{array}$

No. half-yards in a rod, $11 \overline{)8056}$ half-yds.

No. rods in a furlong, $40 \overline{)732}$ rds. + $\frac{3}{4} = 2$ yds.

No. furlongs in a mile, $8 \overline{)18}$ fur. + 12 rds.
 $\begin{array}{r} 2 \\ \underline{2} \end{array}$ mi. + 2 fur.

Hence, 145025 inches = 2 mi. 2 fur. 12 rds. 2 yds. 1 ft. 5 in.

3. Reduce 3 fur. 12 rds. 2 yds. 2 ft. 4 in. to inches.

4. Reduce 7256381 inches to higher denominations.

5. Reduce 8356 in. to links.

6. Reduce 132 ft. to chains.

7. Reduce 66 links to feet.

8. Reduce $5\frac{1}{2}$ yds. to links.

197. The measure of areas is called **SQUARE MEASURE**, and is used in measuring surfaces.

198. The *unit* is a *square one foot on each side*.

SQUARE MEASURE.

144 Square Inches.....	make 1 Square Foot.	<i>sq. ft.</i>
9 Square Feet.....	1 Square Yard.	<i>sq. yd.</i>
$30\frac{1}{4}$ Square Yards.....	1 Square Rod.	<i>sq. rd.</i>
40 Square Rods..	1 Rood.	<i>R.</i>
4 Roods.....	1 Acre.	<i>A.</i>
640 Acres.....	1 Square Mile.	<i>sq. mi.</i>

SURVEYORS' SQUARE MEASURE.

625 Square Links.....	make 1 Pole.	<i>P.</i>
16 Poles.....	1 Square Chain.	<i>sq. ch.</i>
10 Square Chains.....	1 Acre.	<i>A.</i>
640 Acres.....	1 Square Mile.	<i>sq. mi.</i>
36 Square Miles.....	1 Township.	<i>T.</i>

199. The measure of any surface is the number of times it contains the unit, and is found by multiplying its length by its breadth.

Example 1. — Reduce 2 A. 1 R. 1 sq. rd. 1 sq. yd. 1 sq. ft. 1 sq. in. to square inches.

Solution. — 2 A. 1 R. 1 sq. rd. 1 sq. yd. 1 sq. ft. 1 sq. in.

4	No. roods in an acre.
8	No. of roods in 2 acres.
1	
9	No. roods in 2 acres, 1 rood.
40	No. sq. rds. in a rood.
360	No. sq. rds. in 9 roods.
1	
361	No. sq. rds. in 9 roods, 1 sq. rd.
80 $\frac{1}{4}$	No. sq. yds. in a sq. rod.
10830	
90 $\frac{1}{4}$	
10920 $\frac{1}{4}$	No. sq. yds. in 361 sq. rds.
1	
10921 $\frac{1}{4}$	No. sq. yds. in 361 sq. rds. 1 sq. yd.
9	No. sq. ft. in a sq. yd.
98289	
2 $\frac{1}{4}$	
98291 $\frac{1}{4}$	No. sq. ft. in 10921 $\frac{1}{4}$ sq. yds.
1	
98292 $\frac{1}{4}$	No. sq. ft. in 10921 $\frac{1}{4}$ sq. yds. 1 sq. ft.
144	No. sq. in. in a sq. ft.
893168	
393168	
98292	
86	
14154084	No. sq. in. in 98292 $\frac{1}{4}$ sq. ft.
1	
14154085	No. sq. in. in 98292 $\frac{1}{4}$ sq. ft. 1 sq. in.

2. Reduce 14154085 sq. in. to higher denominations.

3. Find the cost of 2 A. 3 sq. yds. 4 sq. ft., at 5 cents per sq. ft.

4. Reduce 613474 sq. ft. to higher denominations.

5. Reduce 10 sq. ch. to sq. yds.

6. Reduce 4 R. to sq. links.

7. How many sq. rds. in $\frac{3}{4}$ of a township?

8. Reduce 1 sq. mi. 100 A. to sq. in.

9. Reduce 1 sq. mi. 100 A. to P.

CUBIC MEASURE.

200. CUBIC MEASURE is used in finding the *solid contents* of bodies having extension in three directions, *length*, *breadth*, and *thickness*.

201. A *cube* is a *solid* bounded by six equal square surfaces.

202. The *unit of measure* is a small cube one inch in length, one inch in breadth, one inch in thickness; which contains one cubic inch.

203. The *numerical measure* of any large cube is the number of times it contains this unit. Hence, the cubical contents of any cube is found by taking the continued product of its three dimensions.

Illustration.—The cubical contents of a solid 4 inches on each edge, is 4 times 4 times 4, or 64 cubic inches.

CUBIC MEASURE.

1728 Cubic Inches.....	make 1 Cubic Foot.	<i>cu. ft.</i>
27 Cubic Feet.....	1 Cubic Yard.	<i>cu. yd.</i>
24 $\frac{1}{2}$ Cubic Feet.....	1 Perch,	<i>pch.</i>

WOOD MEASURE.

16 Cubic Feet.....	make 1 Cord Foot.	<i>cd. ft.</i>
8 Cord Feet.....	1 Cord	<i>cd.</i>
128 Cubic Feet.....	1 Cord.	

204. The perch is used in measuring stone-work, or masonry, which is generally estimated by the perch.

Example 1. — Reduce 3 cu. yds. 3 cu. ft. 3 cu. in. to cu. inches.

Solution. — 3 cu. yds. 3 cu. ft. 3 cu. in.

27 No. cu. ft. in a cu. yd.

81 No. cu. ft. in 3 cu. yds.

3

84 No. cu. ft. in 3 cu. yds, 3 cu. ft.

1728 No. cu. in. in a cu. ft.

6912

13824

145152 No. cu. in. in 84 cu. ft.

3

145155 No. cu. in. in 84 cu. ft., 3 cu. in.

2. Reduce 145155 cu. in. to higher denominations.

3. How many cu. yds. in 27648 cu. in.?

4. How many cu. ft. in $6\frac{3}{4}$ cords? In $12\frac{1}{4}$ cords?

5. Change 471837 cu. in. to cu. yds.

6. Change 37 cd. ft. to cu. in.

205. The most important measures of capacity are

a. DRY MEASURE.

b. LIQUID MEASURE.

206. **Dry Measure** is used in measuring grain, fruit, etc.

The denominations used are *pint*, *quart*, *peck*, *bushel*.

TABLE FOR DRY MEASURE.

2 Pints	make 1 Quart.	<i>pt. qt.</i>
8 Quarts.....	1 Peck.	<i>pk.</i>
4 Pecks	1 Bushel.	<i>bu.</i>

The cubic contents of the Winchester bushel is 2150.4 *cubic inches*.

207. To change any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. How many cubic inches in 1 peck? in 2 pecks?
2. How many cubic inches in 1 quart? in 2 quarts?
3. How many cubic inches in 1 pint? in 2 pints?
4. Change 2 quarts to pints; 5 quarts to pints.
5. Change 7 quarts to pints; 10 quarts to pints.
6. Reduce 2 pecks to quarts; to pints.
7. Reduce 5 pecks to quarts; to pints.
8. Reduce 7 pecks, 5 quarts, to quarts; to pints.
9. Reduce 10 pecks, 7 quarts, 3 pints, to pints.
10. Reduce 2 bushels, 2 pecks, 4 quarts, 1 pint, to pints.

208. To reduce any number of units of one denomination to an equivalent value expressed in terms of any higher denomination.

1. Reduce 10 pints to quarts; 20 pints to quarts.
2. Reduce 32 pints to quarts; to pecks.

3. Reduce 64 pints to quarts ; to pecks ; to bushels.
4. Reduce 100 pints to higher denominations.
5. Reduce 100 quarts to higher denominations.
6. Reduce 1000 quarts to higher denominations.
7. Reduce 1000 pints to higher denominations.
8. How many bushels of wheat will a bin contain that is 10 feet long, 5 feet wide, 4 feet deep?

LIQUID MEASURE.

209. Liquid Measure is generally used for measuring all kinds of liquids.

The denominations used are gill, pint, quart, gallon, barrel, hogshead.

TABLE FOR LIQUID MEASURE.

4 Gills.....	make 1 Pint.	<i>qt. pt.</i>
2 Pints.....	1 Quart.	<i>qt.</i>
4 Quarts.....	1 Gallon.	<i>gal.</i>
31½ Gallons.....	1 Barrel.	<i>bbl.</i>
63 Gallons.....	1 Hogshead.	<i>hhd.</i>

NOTE.— *Tierce*, *pipe* and *tun* are vessels of varying capacity, which generally contain, respectively, 42 gallons, 126 gallons, 252 gallons.

The wine gallon contains 231 cubic inches.

The beer gallon contains 282 cubic inches.

EQUIVALENT.

1 bu. equals 9.31 wine gal., nearly.

1 bu. equals 7.62½ beer gal., nearly.

210. To reduce any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. Reduce 2 pints to gills ; 5 pints to gills.
2. Reduce 7 pints, 2 gills, to gills.
3. Change 2 gallons to quarts ; to pints ; to gills.
4. Reduce 2 barrels to quarts ; to pints ; to gills.
5. 3 hhds. contain how many pints ?

211. To reduce any number of units of one denomination to an equivalent value expressed in terms of higher denominations.

EXAMPLES.

1. Change 72 gills to pints ; to quarts.
2. Reduce 100 gills to higher denominations.
3. Change 1000 pints to higher denominations.
4. In 720 pints, how many hhds. ?
5. In 471 gills there are how many barrels.

212. Dry and Liquid Measure compared.

EXAMPLES.

1. If a man should buy a bushel of strawberries by beer measure, and sell them by wine measure, how many quarts would he gain ?

Suggestion. — In 1 *bu.* there are how many *qts.* wine measure?
In 1 *bu.* there are how many *qts.* beer measure ?

2. How much would be gained, or lost, by buying a barrel of vinegar by beer measure, and selling it by wine measure, at 40 cents per gallon ?

3. What would be gained, or lost, by selling one peck

of currants, at 7 cents a pint wine measure, that cost $6\frac{1}{2}$ cents a pint dry measure?

213. Angular Measure.

The denominations used are *circumference, sign, degree, minute, second*.

TABLE FOR ANGULAR MEASURE.

60 Seconds.....	make 1 Minute.	"	'
60 Minutes.....	1 Degree.		°
30 Degrees.....	1 Sign.		s.
12 Signs.....	1 Circumference.		C.

EXAMPLES.

1. Change 12 signs to degrees ; to minutes ; to seconds.
2. Reduce 5 signs, 5 degrees, to minutes.
3. Change 3 signs, 3 degrees, 3 minutes, to seconds.
4. Reduce 10 signs, 10 degrees, 10 minutes, 10 seconds, to seconds.

MEASURE OF DURATION.

214. Time is a part of duration, and for purposes of calculation is measured by the revolution of the Earth on its axis.

One such revolution determines the length of our day, while one revolution of the Earth about the Sun constitutes our year.

215. The year is subdivided into seasons, known as Spring, Summer, Autumn, Winter.

The year is also divided into twelve months, each consisting of a certain number of days.

Although the seasons consist of the same number of months, the months do not consist of the same number of days.

Spring Months.	Summer Months.	Autumn Months.	Winter Months.
March,	June,	September,	December,
April,	July,	October,	January,
May.	August.	November.	February.

LENGTH OF THE MONTHS.

Months having 30 days.	{ April, June, September, November.	Months having 31 days.	{ January, March, May, July, August, October, December.
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NOTE. — In leap year February has 29 days; but in common years it has only 28 days.

a. Every year, except centennial, that is divisible by 4, is a leap year.

b. Every centennial year that is divisible by 400 is a leap year.

The subdivisions of the month, day, hour, minute, are given in the table below.

TIME TABLE.

60 Seconds	make 1 Minute.	sec.	mi.
60 Minutes	1 Hour.		hr.
24 Hours	1 Day.		da.
365 Days	1 Year.		yr.
366 Days	1 Leap Year.		
7 Days	1 Week.		wk.
4 Weeks	1 Lunar Month.		mo.
12 Calendar Months	1 Year.		
52 Weeks	1 Year.		
100 Years	1 Century.		C.

NOTE. — This division of time is not strictly accurate, but gives rise to an error of 1 day in 3600 years.

216. To reduce any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. Change 2 hours to minutes ; 5 minutes to seconds.
2. Reduce 1 year to hours ; to minutes ; to seconds.
3. How many hours in the month of May?
4. How many more hours has July than February in leap year?

217. To reduce any number of units of one denomination to an equivalent value expressed in terms of any higher denomination.

EXAMPLES.

1. Reduce 100 seconds to hours, minutes and seconds.
2. Reduce 1000 minutes to hours and minutes.
3. Reduce 10000 seconds to higher denominations.

TROY WEIGHT.

218. Troy Weight is used in weighing gold, silver, precious stones, and in philosophical experiments.

The denominations used are *grain*, *pennyweight*, *ounce*, *pound*, and sometimes the *carat*, which is used in determining the purity of gold, 24 carats fine being *pure* gold.

TABLE FOR TROY WEIGHT.

24 Grains.....	make 1 Pennyweight.	<i>gr.</i>	<i>pwt.</i>
20 Pennyweights.....	1 Ounce.		<i>oz.</i>
12 Ounces.....	1 Pound.		<i>lb.</i>

219. To reduce any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. How many grains are there in 1 pound? in 3 pounds?

2. Change 1 pound, 1 ounce to pennyweights.

3. Change 2 pounds, 2 ounces, 2 pennyweights, to grains.

4. Reduce 10 pounds to grains.

5. Reduce 1 lb., 2 oz., 3 pwts., 4 grs., to grains.

220. To reduce any number of units of one denomination to an equivalent value expressed in terms of any higher denomination.

EXAMPLES.

1. Change 240 grains to pennyweights.

2. Change 2400 grains to ounces.

3. Change 120 pwts. to pounds.

4. Reduce 100 grains to higher denominations.

5. Reduce 1000 grains to higher denominations.

APOTHECARIES WEIGHT.

221. Apothecaries Weight is used in preparing medical prescriptions.

The denominations used are *grain*, *scruple*, *dram*, *ounce*, *pound*.

The grain, the ounce, the pound, are the same as in Troy weight.

The pound in each of these weights consists of 5760 grains.

TABLE FOR APOTHECARIES WEIGHT.

20 Grains.....	make 1 Scruple.	gr.	℥.
8 Scruples	1 Dram.		ʒ.
8 Drams	1 Ounce.		℥.
12 Ounces	1 Pound.		lb.

222. To reduce any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. Reduce 1 pound to ounces ; to drams ; to scruples ; to grains.
2. Reduce 1 lb. 2℥, 3℥, 2℥, 5 grs. to grains.
3. Reduce 2℥, 5℥, 2℥ to grains.
4. Change 5 pounds to grains.
5. Change 7 drams to grains.

223. To reduce any number of units of one denomination to an equivalent value expressed in terms of any higher denomination.

EXAMPLES.

1. Change 5760 grains to higher denominations.
2. Change 100 grains to higher denominations.
3. Reduce 1000 grains to higher denominations.
4. Reduce 1000 scruples to higher denominations.
5. Reduce 10000 ℥ to higher denominations.

AVOIRDUPOIS WEIGHT.

224. Avoirdupois Weight is used in weighing coarse and heavy articles.

The denominations used are *dram*, *ounce*, *pound*, *hundredweight*, *ton*. Sometimes *quarter* and *long ton* are used.

TABLE FOR AVOIRDUPOIS WEIGHT.

16 Drams.....	make 1 Ounce.	dr.	oz.
16 Ounces.....	1 Pound.		lb.
25 Pounds	1 Quarter.		qr.
4 Quarters.....	1 Hundredweight.		cwt.
20 Hundredweight.....	1 Ton.		T.

NOTE. — In the *long* or *gross* ton 28 pounds make a quarter, used in the custom-house, and in weighing coal.

225. To reduce any number of units of one denomination to an equivalent value expressed in terms of any lower denomination.

EXAMPLES.

1. Change 1 cwt. to oz. ; to drams.
2. Change 3 qrs. to oz. ; to drams.
3. Change 1 long ton to drams.
4. Change 1 ton, 2 cwts., 3 qrs., 4 lbs., 5 oz., 6 drs., to drams.
5. Reduce 10 tons to drams.

226. To reduce any number of units of one denomination to an equivalent value expressed in terms of any higher denomination.

EXAMPLES.

1. Reduce 7000 drams to higher denominations.
2. Reduce 5670 drams to higher denominations.
3. Reduce 100 drams to higher denominations.
4. Reduce 1000 drams to higher denominations.
5. Reduce 1200 drams to higher denominations.

AVOIRDUPOIS AND TROY WEIGHT COMPARED.

227. The **grain** which is the basis of both these weights is the **basis** of comparison, there being in a pound, Troy weight, 5670 grains, and in the pound Avoirdupois, 7000 grains.

EXAMPLES.

1. In 5 pounds Troy, how many pounds of Avoirdupois?

Suggestion. — In 5 lbs. Troy, how many grains?

In 28350 grains, how many pounds Avoirdupois?

2. In 5 pounds Avoirdupois, how many pounds Troy?

Suggestion. — In 5 lbs. Avoirdupois, how many grains?

In 35000 grains, how many pounds Troy?

3. How much does a physician gain who buys 10 pounds of opium by Avoirdupois, and sells it by Apothecaries weight.

4. A man bought sugar at $6\frac{1}{4}$ cents per pound Avoirdupois, and sold it at the rate of 6 cents a pound Troy weight. Would he make or lose, and how much on 300 pounds?

5. How many pounds Troy are there in 256 ounces Avoirdupois weight?

METRIC SYSTEM.

228. The **Metric System** originated in France.

229. The **Primary** or **Base Unit** of this system of weights and measures, is the **Metre**.

230. The **Metre** is a linear unit whose length was determined by measuring, on the meridian passing through Barcelona and Dunkirk, the distance from the equator to the pole. One ten-millionth part of this distance, which is nearly 39.37 inches, is the length of the **Metre**.

231. From the **Metre**, as the primary unit of the system, three other principal units, and one subordinate unit, are derived.

232. These three derivative units are :

The **Are**, the unit of surface.

The **Litre**, the unit of volume.

The **Gramme**, the unit of weight.

233. The **Are** is a square whose side is ten **Metres**.

The **Litre** is a cube whose edge is one-tenth of a **Metre**.

The **Gramme** is a volume of rain water equal to a cube whose edge is one-hundredth of a **Metre**.

The **Stere** has a special application in the measurement of volumes. Art. 244, note.

LINEAR MEASURE.

234. The unit of measure, in determining distances, is the **Metre**.

In this measure there are four multiples and three sub-multiples of the base unit.

The multiples of the **Metre** are, decametre, hectometre, kilometre, myriametre.

The sub-multiples are decimetre, centimetre, millimetre.

235. The **scale** is uniform, being **ten**, the same as in whole numbers.

The **Millimetre** is the **lowest**, and the **Myriametre** is the **highest** denomination.

NOTE.—Let the pupils arrange these several denominations in the form of a table.

EXAMPLES.

1. Change one metre to feet.
2. How many inches in 1 decimetre? 1 centimetre? 1 millimetre?
3. How many feet and inches in 1 decametre?
4. Reduce 1 hectometre to rods, feet, and inches. 1 kilometre.
5. Change 1 myriametre to mi., fur., rds., ft., and inches.

SQUARE MEASURE.

236. The **unit** of measure in determining surfaces is the **Are**.

In this measure there is one multiple, and one sub-multiple of the base unit.

The multiple of the **Are** is the **Hectare**.

The sub-multiple of the **Are** is the **Centiare**.

237. The **scale** is uniform, being one hundred.

NOTE.—Let the pupil construct the table for this measure.

238. To find the numerical value of the base unit in this measure.

Explanation.—By the definition, the are is a square ten metres on each side; hence, it contains 100 square metres.

A square metre equals (39.37×39.37) , or 1549.9969 sq. in.

Then, 100 sq. metres contain 154999.69 sq. in.

But 154999.69 sq. in. equal 119.59 sq. yds.

Hence, the *Are* equals 119.6 (nearly) sq. yds., or nearly 4 perches.

239. The problems under this measure are :

1. To reduce ares to centiares.
2. To reduce ares to hectares.
3. To reduce centiares to ares.
4. To reduce centiares to hectares.
5. To reduce hectares to ares.
6. To reduce hectares to centiares.
7. To change any number of units in the Metric System to an equivalent value in the common system.
8. To change any number of units in the common system to an equivalent value in the Metric System.

NOTE.—Let the pupils make and solve an example under each case.

MEASURES OF CAPACITY.

240. Measures of capacity are :

1. Cubic Measure.
2. Dry Measure.
3. Liquid Measure.

241. The unit of measure in determining volumes is the **Litre**. In this measure there are **three** multiples and **three** sub-multiples of the base unit.

The multiples of the base unit are, *decalitre*, *hectolitre*, *kilolitre*.

The sub-multiples of the base unit are *decilitre*, *centilitre*, *millilitre*.

The **Millilitre** is the lowest, and the **Kilolitre** the highest, denomination.

The scale is uniform, being *ten*.

NOTE. — Let the pupils construct the table for this measure.

242. To find the numerical value of the base unit in this measure.

Explanation. — By the definition, the litre is a cube whose edge is *one-tenth* of a metre.

The cube of one-tenth of a metre is 61.023378 cubic inches.

Then, 1st. A litre equals $\frac{61.023378}{2150.4}$ bushels, or $\frac{61.023378 \times 4 \times 8 \times 2}{2150.4}$
 $= 1.81606$ pints, dry measure.

And, 2d. A litre equals $\frac{61.023378}{231}$ gallons, or $\frac{61.023378 \times 4}{231} = 1$ qt.
 1.1335 pts., wine measure.

243. To reduce any number of units expressed in the Metric System to an equivalent value expressed in terms of the common system.

1. Express 2 bu. dry measure in terms of the metric system.
2. Express 3 gal. in terms of the metric system.
3. Express 5 gal. 1 qt. in terms of the metric system.

244. To reduce any number of units expressed in the common system to an equivalent value expressed in terms of the metric system.

1. Change 12 litres to dry measure.
2. Change 15 litres to liquid measure.
3. Express 10 bbls. in terms of the metric system.

NOTE. — The cubic metre is called the **Stere**, when used in measuring wood or stone, and equals .2759 of a cord, or 1.308 cubic yards.

AVOIRDUPOIS AND TROY WEIGHTS.

245. The unit of weight is the Gramme.

In these weights there are **six** multiples of the base unit and **three** sub-multiples.

The multiples of the base unit are, *decagramme*, *hectogramme*, *myriagramme*, *quintal*, *millier*, or *tonneau*.

The sub-multiples of the base unit are *decigramme*, *centigramme*, *milligramme*.

The **lowest** of these denominations is the **Milligramme**, and the **highest**, *millier*, or **Toneau**.

246. The scale is uniform, being *ten*.

NOTE. — Let the pupils construct the table.

247. To find the numerical value of the gramme in ounces Avoirdupois.

Explanation.—*a.* By the definition, the gramme equals in weight *one-millionth* of a cubic metre of rain water.

b. A cubic metre contains 35.314454 cubic feet.

c. A cubic foot of rain water weighs 1000 ounces Av. Hence, a cubic metre weighs 35314.454 ounces Av., or a gramme weighs .035 oz. Av.

248. To express Avoirdupois weight in terms of the Metric System.

Explanation. — This being the reverse of 247, find an equivalent value in ounces avoirdupois.

Then find an equivalent value in grammes.

249. TABLE OF EQUIVALENTS.

A Metre, measure of length,

	Equals 39.37 inches.		{ 1 inch equals .0254 metre.
a. {	Or 3.28 ft.	b. {	1 foot " .3048 metre.
	Or 1.0936 yds.		1 yard " .9144 metre.

An Are, measure of surfaces,

	Equals 119.6 sq. yds.		{ 1 sq. yd. equals .834 centiare.
a. {	Or 8.953 sq. rds.	b. {	1 sq. rd. " .2529 are.

A Litre, measure of capacity,

	Equals 61.02 cu. inches.		{ 1 cubic inch equals .0163 litre.
a. {	Or .908 q. D. M.	b. {	1 quart D. M. " 1.1013 litre.
	Or 1.056 " L. M.		1 quart L. M. " .946 litre.

A Gramme, measure of weight,

	Equals 15.432 grs. T. W.		{ 1 grain equals .0648 gramme.
a. {	Or .035 oz. Av. W.	b. {	

A Stere, measure of solids,

	Equals 35.3144 cu. ft.		{ 1 cu. ft. equals .0283 stere.
a. {	Or 1.3079 cu. yds.	b. {	1 cu. yd. " .764 stere.
	Or .2759 cords.		1 cord " 3.62 stere.

NOTE.—The above equivalents are never given in *exact* values. Only approximate values are possible in most cases.

250. Addition of Compound Numbers.

In adding compound numbers we must observe how many units of the denomination added make *one* unit of the next higher denomination.

In this respect alone the addition of compound numbers differs from the addition of simple numbers.

ADD			ADD				ADD				
£	s.	d.	bu.	pk.	qt.	pt.	wk.	da.	hr.	mi.	sec.
4	9	3	24	2	5	2	2	5	21	40	16
5	16	8	11	1	3	0	1	7	20	43	38
9	16	7	15	1	5	1	3	1	17	56	51
3	12	9	7	2	4	5	5	6	16	19	18
23	15	3									

ADD				ADD							
lbs.	oz.	pwt.	grs.	sq. m.	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.	
19	9	20	16	11	120	2	12	7	2	131	
16	8	19	20	34	360	2	35	6	5	99	
1	11	0	6	32	386	3	14	3	4	123	
8	3	15	14	18	587	2	12	8	0	141	
1	7	17	19	12	150	1	0	0	3	92	

ADD					ADD						ADD		
lb.	3	3	3	gr.	mi.	fur.	rd.	yd.	ft.	in.	cu. yd.	cu. ft.	cu. in.
9	2	5	0	10	12	7	39	5	2	10	20	29	1205
1	0	2	1	15	11	6	38	4	1	9	19	30	1250
12	1	7	2	19	10	5	37	3	0	8	18	31	1125
10	4	5	0	13	9	4	36	2	1	7	17	32	1052

ADD					ADD				
T.	cwt.	qr.	lb.	oz.	C.	s.	°	'	"
65	15	3	21	9	100	11	59	49	39
64	16	2	22	8	99	9	49	39	29
63	17	3	23	7	19	10	39	29	9

251. Subtraction of Compound Numbers.

In subtracting one compound number from another we must observe how many units make *one* unit of the next higher denomination.

In this respect alone subtraction in compound numbers differs from subtraction in simple numbers

(1.)				(2.)				(3.)			
£	s.	d.	far.	gal.	qt.	pt.	gi.	bu.	pk.	qt.	pt.
From 22	19	2	3	From 35	2	1	0	From 15	2	2	0
Take 14	12	10	2	Take 31	3	1	3	Take 12	3	3	1
Rem.	8	6	4 1	Rem.				Rem.			

(4.)						(5.)			
T.	cwt.	qr.	lb.	oz.	dr.	lb.	oz.	pwt.	gr.
From 41	19	3	14	15	13	From 45	9	11	18
Take 40	19	3	15	15	14	Take 22	10	12	19
Rem.						Rem.			

(6.)					(7.)				
lb.	3	3	3	gr.	wk.	da.	hr.	min.	sec.
From 14	9	5	2	12	From 34	2	15	59	49
Take 11	8	6	2	13	Take 33	5	21	59	51
Rem.					Rem.				

(8.)					(9.)				
sq.mi.	A.	B.	sq.rd.	sq.yd.	C.	s.	°	'	"
From 26	121	3	39	30	From 15	11	5	55	58
Take 24	120	2	39	31	Take 12	11	6	56	59
Rem.					Rem.				

(10.)							(11.)						
	<i>mi.</i>	<i>fur.</i>	<i>rd.</i>	<i>yd.</i>	<i>ft.</i>	<i>in.</i>		<i>A.</i>	<i>R.</i>	<i>sq.rd.</i>	<i>sq.yd.</i>	<i>sq.ft.</i>	<i>sq.in.</i>
From	21	7	21	4	2	9	From	10	2	30	19	7	121
Take	15	6	31	5	3	11	Take	9	2	29	19	6	132
Rem.							Rem.						
(12.)							(13.)						
	<i>cu.yd.</i>	<i>cu.ft.</i>	<i>cu.in.</i>					<i>mi.</i>	<i>ch.</i>	<i>rd.</i>	<i>li.</i>		
From	123	14	1128				From	1	5	7	12		
Take	122	15	1129				Take		6	8	15		
Rem.							Rem.						

252. Multiplication of Compound Numbers.

In multiplying, as in adding or subtracting compound numbers, we must observe how many units of the denomination multiplied, make *one* unit of the next higher denomination.

In this respect alone multiplication in compound numbers differs from multiplication in simple numbers.

(1.)					(2.)					
	£	s.	d.	far.		A.	R.	sq.rd.	sq.yd.	sq.ft.
Multiply	21	19	11	3	Multiply	3	2	13	27	6
By	2				By	3				
Prod.	43	19	11	2	Prod.					
(3.)					(4.)					
	C.	s.	°	'	"		qt.	pt.	gi.	
Multiply	15	11	59	59	59	Multiply	8	3	2	
By	4					By	5			
Prod.						Prod.				

$$\begin{array}{r}
 \text{(5.)} \\
 \text{bu.} \quad \text{pk.} \quad \text{qt.} \\
 \text{Multiply } 7 \quad 3 \quad 7 \\
 \text{By} \quad \quad \quad 6 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(6.)} \\
 \text{bu.} \quad \text{pk.} \quad \text{qt.} \\
 \text{Multiply } 9 \quad 3 \quad 7 \\
 \text{By} \quad \quad \quad 7 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(7.)} \\
 \text{gal.} \quad \text{qt.} \quad \text{pt.} \\
 \text{Multiply } 3 \quad 2 \quad 1 \\
 \text{By} \quad \quad \quad 8 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(8.)} \\
 \text{qt.} \quad \text{pt.} \quad \text{gi.} \\
 \text{Multiply } 3 \quad 1 \quad 3 \\
 \text{By} \quad \quad \quad 9 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(9.)} \\
 \text{mi. fur. rd. yd. ft. in.} \\
 \text{Multiply } 3 \quad 5 \quad 17 \quad 12 \quad 10 \quad 10 \\
 \text{By} \quad \quad \quad \quad \quad 10 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(10.)} \\
 \text{lb.} \quad \text{3} \quad \text{3} \quad \text{3} \quad \text{gr.} \\
 \text{Multiply } 5 \quad 7 \quad 5 \quad 2 \quad 9 \\
 \text{By} \quad \quad \quad \quad \quad 11 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(11.)} \\
 \text{T. cwt. gr. lbs. oz. dr.} \\
 \text{Multiply } 7 \quad 4 \quad 3 \quad 24 \quad 15 \quad 14 \\
 \text{By} \quad \quad \quad \quad \quad 12 \\
 \hline
 \text{Prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{(12.)} \\
 \text{wk. da. hr. min. sec.} \\
 \text{Multiply } 3 \quad 4 \quad 5 \quad 6 \quad 7 \\
 \text{By} \quad \quad \quad \quad \quad 13 \\
 \hline
 \text{Prod.}
 \end{array}$$

253. Division of Compound Numbers.

In the division of compound numbers we must observe how many units one unit of the denomination divided makes of the next *lower* denomination.

In this respect alone division in compound numbers differs from division in simple numbers.

(1.)

Divide 7 cwt., 1 qr., 23 lbs., 7 oz. by 4.

Explanation.

One-fourth of 7 cwt. is 1 cwt., with a remainder of 3 cwt. Reducing 3 cwt. to qrs. and adding the 1 qr., the new dividend is 13 qrs.

One-fourth of 13 qrs. is 3 qrs., with a remainder of 1 qr. Reducing the 1 qr. to pounds and adding the 23 pounds, the new dividend is 48 pounds.

One-fourth of 48 lbs. is 12 lbs.

One-fourth of 7 oz. is 1 oz., with a remainder of 3 oz. Reducing the 3 oz. to drams the new dividend is 48 drs.

One-fourth of 48 drs. is 12 drs.

Hence, one-fourth of 7 cwt., 1 qr., 23 lbs., 7 oz., is 1 cwt., 3 qrs., 12 lbs., 1 oz. 12 drs.

OPERATION.

	<i>cwt.</i>	<i>qr.</i>	<i>lbs.</i>	<i>oz.</i>	<i>drs.</i>
4)	7	1	23	7	0
	1	3	12	1	12

In a similar manner perform the operations indicated in the following examples :

(1.)

$$\begin{array}{r} \text{mi. rd. ft.} \\ 6 \overline{) 57 \ 4 \ 12} \\ \text{Quo.} \quad \underline{9 \ 54 \ 2} \end{array}$$

(2.)

$$\begin{array}{r} \text{A. sq. rd. sq. yd. sq. ft. sq. in.} \\ 12 \overline{) 34 \ 32 \ 8 \ 5 \ 48} \\ \text{Quo.} \end{array}$$

(3.)

$$\begin{array}{r} \text{mi. rd. ft.} \\ 3 \overline{) 20 \ 25 \ 15} \\ \text{Quo.} \end{array}$$

(4.)

$$\begin{array}{r} \text{cd. cd. ft. cu. ft. cu. in.} \\ 9 \overline{) 43 \ 4 \ 11 \ 1512} \\ \text{Quo.} \end{array}$$

(5.)

$$\begin{array}{r} \text{T. cwt. lb. oz.} \\ 7 \overline{) 3 \ 4 \ 6 \ 12} \\ \text{Quo.} \end{array}$$

(6.)

$$\begin{array}{r} \text{wk. da. hr. mi. sec.} \\ 4 \overline{) 3 \ 2 \ 5 \ 35 \ 49} \\ \text{Quo.} \end{array}$$

(7.)					(8.)							
£	s.	d.	far.	sq. mi.	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.		
5)	17	-9	8	1	6)	121	131	3	31	13	2	140
Quo.					Quo.							

NOTE. — *a.* The pupils should prove each example in subtraction, by making it an example in addition of compound numbers.

b. Each example in multiplication should be verified by division, and each example in division should be verified by multiplication.

c. Each step in the process should be fully and clearly explained by the pupils as they proceed with the operation.

d. To understand an explanation given in the book, or by the teacher, and to be able to give the explanation, unaided by the teacher, are quite different things. No pupil should be satisfied to simply understand what is explained to him; he ought to be able to give a good explanation himself, of every example or problem in a lesson, before leaving it.

e. Having acquired facility in performing the merely mechanical processes, and having mastered the logic of the process, before leaving the subject, the pupil should write out carefully a rule for performing the process.

REDUCTION OF DENOMINATE FRACTIONS.

254. A **denominate fraction** is one which expresses one or several of the equal parts of a denominate number.

255. A denominate fraction may be changed from

a. One of a higher to one of a lower denomination.

b. One of a lower to one of a higher denomination.

c. A fraction to integers of lower denominations.

d. Integers of a lower to a fraction of a higher denomination.

256. To reduce a denominate fraction from a higher to one of a lower denomination.

1. Reduce $\frac{3}{8}$ of an hour to an equivalent fraction of a lower denomination.

Suggestion. — What part of 1 minute is $\frac{3}{8}$ of 60 minutes?
 What part of 1 second is $\frac{3}{8}$ of 60 seconds?
 Then, $\frac{3}{8}$ of an hour is what part of a minute?
 Then, $\frac{3}{8}$ of an hour is what part of a second?

2. Reduce $\frac{2}{3}$ of a bushel to the fraction of a peck.

3. Reduce $\frac{5}{8}$ of a peck to the fraction of a pint.

257. To reduce a denominate fraction from a lower to one of a higher denomination.

1. Reduce $\frac{1}{2}$ of a pint to the fraction of a peck.

Suggestion. — $\frac{1}{2}$ of a pint is what part of a quart, or 2 pints?
 $\frac{1}{4}$ of a quart is what part of a peck, or 8 quarts?
 Then, $\frac{1}{2}$ of a pint is what part of a peck?

2. Reduce $\frac{2}{3}$ of a pint to a fraction of a gallon.

3. Reduce $\frac{2}{3}$ of an inch to a fraction of a yard.

258. To reduce a denominate fraction to integers of a lower denomination.

1. Reduce $\frac{3}{4}$ of a sq. yd. to integers.

Suggestion. — $\frac{3}{4}$ of a sq. yd. is $\frac{3}{4}$ of 9 sq. ft., or $6\frac{3}{4}$ sq. ft.
 $\frac{3}{4}$ of a sq. ft. is $\frac{3}{4}$ of 144 sq. in.
 Then, $\frac{3}{4}$ of a sq. yd. equals what?

1. Reduce $\frac{1}{6}$ of a mile to integers.

2. Reduce $\frac{3}{4}$ of a furlong to integers.

3. Reduce $\frac{2}{3}$ of an acre to integers.

4. Reduce $\frac{4}{5}$ of a rod to integers.

259. To reduce a denominate number to an equivalent fraction of a higher denomination.

1. Change '2 feet 3 inches to the decimal of a yard.

Suggestion.—3 inches is what decimal of a foot?

2.25 ft. is what part of a yard?

Why divide by 3?

2. Change 2 pints to the fraction of a peck.
3. Change 3 pints to the fraction of a bushel.
4. Change 8 sq. rds. to the fraction of an acre.
5. Reduce $\frac{1}{2}$ of a foot to rods.
6. One-eighth of an inch is what part of a rod?
7. One-fifth of a quarter is what part of a ton?
8. One-third of a grain Troy is what part of an ounce?

MISCELLANEOUS EXAMPLES.

1. Reduce $\frac{1}{8}$ of a mile to the fraction of an inch.
2. Reduce $\frac{1}{8}$ of a mile to integers of a lower denomination.
3. Reduce $\frac{7}{8}$ of a pound to the fraction of a ton.
4. Reduce $\frac{7}{8}$ of a £ to the fraction of a farthing.
5. $\frac{7}{8}$ of a £ to integers of a lower denomination.
6. Reduce $\frac{7}{8}$ of a farthing to the decimal of a £.
7. What decimal of a yard is $\frac{5}{8}$ of a foot?
8. What decimal of a day is $\frac{3}{5}$ of a minute?
9. What fraction of a minute is $\frac{3}{5}$ of a day?
10. Reduce $\frac{3}{5}$ of a cwt. to integers of a lower denomination.
11. Reduce $\frac{2}{5}$ of a shilling to lower denominations.
12. Reduce .1357 of a yard to integers.
13. Reduce $\frac{4}{5}$ of a £ to the fraction of a farthing.
14. Reduce .4763 of £ to integers.

15. Reduce $\frac{5}{18}$ of a year to integers.
16. Reduce $\frac{8}{7}$ of a day to integers. $\frac{7}{8}$ of an hour to integers
17. From $\frac{8}{7}$ of a day subtract $\frac{7}{8}$ of an hour.
18. To $\frac{8}{7}$ of a day add $\frac{7}{8}$ of an hour.
19. From $\frac{9}{11}$ of a ton subtract $\frac{9}{11}$ of a cwt.
20. To $\frac{9}{11}$ of a ton add $\frac{9}{11}$ of a cwt.

LONGITUDE AND TIME.

260. Longitude and Time are employed in determining,

- a. *The difference of longitude between two places, when their difference of time is known.*
- b. *The difference of time between two places, when their difference of longitude is known.*

261. The solution of these two problems in longitude and time depends on two facts :

- a. *The fact that the earth revolves on its axis once in twenty-four hours.*
- b. *The fact that 360° make one circumference.*

From these two facts it follows :

- | | |
|--------|---|
| 1st. { | <i>That 1 hour of time corresponds to 15° of longitude.</i> |
| | <i>That 1 minute of time corresponds to 15' of longitude.</i> |
| | <i>That 1 second of time corresponds to 15" of longitude.</i> |

- 2d. { *That 1° of longitude corresponds to 4 minutes of time.*
That 1' of longitude corresponds to 4 seconds of time.
That 1" of longitude corresponds to $\frac{1}{15}$ seconds of time.

This constant relation which subsists between the difference of longitude and the difference of time between any two points on the Earth's surface, enables us to determine

Their difference of time from their difference of longitude.

Their difference of longitude from their difference of time.

262. To reduce Longitude to Time.

1. In 5° of longitude how many minutes of time?

Suggestion. — In 1° there are how many minutes of time?

Then, 5° equal how many minutes of time?

2. In 5' of longitude how many seconds of time?

Suggestion. — 1' of longitude equals how many seconds of time?

Then, 5' of longitude equal how many seconds of time?

3. In 5° 5' of longitude there are how many seconds of time?

4. In 5° 5' 5" of longitude there are how many seconds of time?

263. To Reduce Time to Longitude.

1. Reduce 20 minutes of time to longitude.

Suggestion. — 1 minute of time equals how many minutes of longitude?

Then, 20 minutes of time equal how many minutes of longitude?

2. Reduce 20 seconds of time to longitude.

Suggestion. — 1 second of time equals how many seconds of longitude?

Then, 20 seconds of time equal how many seconds of longitude?

MISCELLANEOUS EXERCISES.

1. The city of Washington is in west long. $77^{\circ} 2' 48''$, and Boston is in west long. $87^{\circ} 4' 9''$. Find their difference in time.

2. At 10 A. M. in Boston, is it earlier or later in Washington.

3. Columbus is $83^{\circ} 3'$ west long.; and when it is 37 min. 33 sec. past 1 P. M., it is 11 o'clock A. M. at San Francisco. Find the longitude of the latter place.

4. Is it always earlier, or later, at all points east of us? Why? At all points west of us? Why?

5. A celestial phenomenon which was observed at the same time at Boston, Greenwich, and St. Petersburg, occurred at 45 min. 46 sec. past 7 o'clock P. M. at Boston, and at 30 min. past midnight following, at Greenwich. Taking the longitude of Greenwich as 0° , and that of St. Petersburg as $30^{\circ} 19'$ east, at what time did it occur at St. Petersburg? What is the longitude of Boston?

DUODECIMALS.

264. Duodecimals are a species of fractions sometimes used in *linear*, *superficial*, or *cubic measure*.

a. The denominator of a common fraction may be *any* number.

b. The denominator of a decimal is 10, or some power of 10.

c. The denominator of a duodecimal is 12 or some power of 12.

265. In decimal fractions, the denominator, though unwritten, is determined by the decimal point.

In duodecimals, the denominator, though unwritten, is determined by certain characters, called indices.

Twelfths of a foot or inches are read *primes*, and marked thus: '.

Twelfths of an inch are read *seconds*, and marked thus: ''.

Twelfths of a second are read *thirds*, and marked thus: '''. .

Twelfths of a third are read *fourths*, and marked thus: ''''.

GENERAL LAWS.

1. *Twelve units of any denomination make one unit of the next higher denomination.*

2. *One unit of any denomination makes twelve units of the next lower denomination.*

Hence, The denominator of *primes* is 12.

The denominator of *seconds* is 144.

The denominator of *thirds* is 1728.

266. All the operations in duodecimals are performed in the same manner as the corresponding operations in compound numbers, generally, the scale in this case being 12.

a. The product of a duodecimal by any abstract number expresses length.

b. The product of two duodecimals expresses superficial contents.

c. The product of three duodecimals expresses cubic contents.

EXAMPLES.

1. Find the superficial contents of the floor of a room 12 ft. 8' in length, and 11 ft. 9' in width.

FIRST OPERATION.

Explanation.—The product of 8' by 9' is 72". Since 12" make 1', 72" make 6'. Multiplying 12 ft. by 9' the product is 108'. Since 12' make 1 ft., 108' make 9 ft. Multiplying 8' by 11 ft. the product is 88'. Since 12' make 1 ft., 88' make 7 ft. 4'. The product of 12 ft. by 11 ft. is 132 ft.; 132 ft. plus 7 ft. gives 139 ft. Finally, 9 ft. 6', and 139 ft. 4' make 148 ft. 10'.

12 ft. 8'	Multiplicand.
11 ft. 9'	Multiplier.
9 ft. 6'	1st. Partial product.
139 ft. 4'	2d. Partial product.
148 ft. 10'	Total product.

SECOND OPERATION.

12 ft. 8' equal..... $12\frac{2}{3}$ ft.
 11 ft. 9' equal..... $11\frac{3}{4}$ ft.

The product of 11 ft. by 12 ft. is.....132 ft. 1st. Partial prod.
 The product of $\frac{2}{3}$ ft. by 11 ft. is..... $7\frac{1}{3}$ ft. 2d. Partial prod.
 The product of 12 ft. by $\frac{3}{4}$ ft. is.....9 ft. 3d. Partial prod.
 The prod. of $\frac{2}{3}$ ft. by $\frac{3}{4}$ ft. is $\frac{1}{2}$ ft., or.. $\frac{1}{2}$ ft. 4th. Partial prod.

 148 $\frac{1}{4}$ ft. Total prod.

2. The area of the floor of a room is 148 ft. 10', and its width is 11 ft. 9'. Find the length of the room.

Explanation.—Since 148 ft. 10', the area of the room, is the product of its length by its width, if we divide 148 ft. 10' by 11 ft. 9' the quotient must be the length of the room. 148 ft., the first term of the dividend, divided by 11 ft., the first term of the divisor, gives 12 ft. the first term of the quotient. The product of 11 ft. 9' by 12 ft. is 141 ft. Subtracting 141 ft. from 148 ft. 10', the remainder is 7 ft. 10' which, reduced to primes, gives 94'. The first term of the divisor is contained in 94' eight primes times. Multiplying 11 ft. 9' by 8' we get for a product 94'. Subtracting 94' from 94' the remainder is zero.

OPERATION.

Divisor.	Dividend.	Quotient.
11 ft. 9')	148 ft. 10'	(12 ft. 8'
	141 ft. 0'	
	7 ft. 10' =	94'
		00

3. Find the product of 8 ft. 11' by 7 ft. 9'.

4. The ceiling of a room is 16 ft. 5' in length, and 13 ft. 7' in width, what is its area?

5. Find the superficial contents of the walls and ceiling of a room whose length is 18 ft. 8', height 9 ft. 9', and width 14 ft. 7'.

6. Find the cubical contents of a block of wood which is 2 ft. 3' long, 1 ft. 11' wide, and 1 ft. 7' thick.

RATIO.**ORAL EXERCISES.**

1. Six is how many times 3?
2. Three is what part of 6?
3. Four is how many times 2?
4. Two is what part of 4?
5. Eight is how many times 4?
6. Four is what part of 8?

In example 1, 2 is the ratio of 6 to 3; in example 2, $\frac{1}{2}$ is the ratio of 3 to 6; in example 3, 2 is the ratio of 4 to 2; in example 4, $\frac{1}{2}$ is the ratio of 2 to 4; in example 5, 2 is the ratio of 8 to 4; in example 6, $\frac{1}{2}$ is the ratio of 4 to 8.

267. Ratio is the quotient arising from the division of one number by another of the same denomination. Thus: The ratio of 12 to 3 is 4; the ratio of 12 horses to 3 horses is 4; the ratio of 12 men to 3 men is 4; the ratio of 12 oz. to 3 oz is 4; the ratio of \$12 to \$3 is 4.

What is the ratio

- | | |
|--|---|
| 1. Of 3 to 12? | 7. Of $7\frac{1}{2}$ to $2\frac{1}{2}$? |
| 2. Of 3 horses to 12 horses? | 8. Of $\frac{2}{3}$ to $1\frac{1}{3}$? |
| 3. Of 3 men to 12 men? | 9. Of $\frac{4}{5}$ to $\frac{3}{5}$? |
| 4. Of 3 oz. to 12 oz.? | 10. Of $\frac{3}{4}$ to $\frac{3}{8}$? |
| 5. Of \$3 to \$12? | 11. Of $\frac{5}{8}$ to $\frac{15}{8}$? |
| 6. Of $2\frac{1}{2}$ to $7\frac{1}{2}$? | 12. Of $3\frac{1}{4}$ to $5\frac{1}{5}$? |

268. The number expressing the ratio of one number to another is always abstract.

269. Ratio is indicated by a colon, thus, 2:14, and is read 2 is to 14. It is sometimes expressed in the form of a fraction, thus, $\frac{2}{14}$.

270. A simple ratio subsists between only two terms, thus, 3:15, or 15:3; 9:18, or 18:9; 21:7, or 7:21.

271. A compound ratio is composed of two or more simple ratios, thus:

$$\begin{array}{ll}
 1. \left\{ \begin{array}{l} 6:12 \\ 7:21 \end{array} \right. & 2. \left\{ \begin{array}{l} 4:5 \\ 7:9 \\ 8:16 \end{array} \right. & 3. \left\{ \begin{array}{l} 2:1 \\ 3:5 \\ 4:7 \\ 6:8 \end{array} \right. & 4. \left\{ \begin{array}{l} 5:15 \\ 15:16 \\ 16:18 \\ 18:24 \\ 24:30 \end{array} \right.
 \end{array}$$

These compound ratios may be expressed thus :

1. $6 \times 7 : 12 \times 21$.
2. $4 \times 7 \times 8 : 5 \times 9 \times 16$.
3. $2 \times 3 \times 4 \times 6 : 1 \times 5 \times 7 \times 8$.
4. $5 \times 15 \times 16 \times 18 \times 24 : 15 \times 16 \times 18 \times 24 \times 30$.

These ratios may be otherwise written, thus :

1. $\frac{6 \times 7}{12 \times 21}$, or $\frac{1}{3}$.
2. $\frac{4 \times 7 \times 8}{5 \times 9 \times 16}$, or $\frac{14}{45}$.
3. $\frac{2 \times 3 \times 4 \times 6}{1 \times 5 \times 7 \times 8}$, or $\frac{3}{35}$.
4. $\frac{5 \times 15 \times 16 \times 18 \times 24}{15 \times 16 \times 18 \times 24 \times 30}$, or $\frac{1}{3}$.

272. The terms of a ratio taken together are called a couplet. The first term of a couplet is the antecedent, the second term the consequent.

273. When a ratio is expressed in a fractional form, the antecedent is the numerator, and the consequent is the denominator.

EXERCISES.

Express the following compound ratios in fractional form, and reduce them to their lowest terms, as in 271.

1. $\left\{ \begin{array}{l} 5 : 20 \\ 15 : 8 \\ 2 : 10 \\ 1 : 6 \end{array} \right.$
2. $\left\{ \begin{array}{l} 6 : 1 \\ 2 : 3 \\ 4 : 12 \\ 7 : 14 \\ 21 : 28 \end{array} \right.$
3. $\left\{ \begin{array}{l} 12 : 15 \\ 14 : 10 \\ 11 : 33 \\ 18 : 52 \\ 27 : 54 \end{array} \right.$
4. $\left\{ \begin{array}{l} 5 : 18 \\ 12 : 9 \\ 14 : 4 \\ 21 : 28 \\ 15 : 25 \\ 32 : 16 \end{array} \right.$

274. The fundamental principles of fractions, 126, apply to ratio.

EXERCISES ON THE FUNDAMENTAL PRINCIPLES.

1. Multiplying the antecedent has what effect on the ratio? Why?

2. Multiplying the consequent has what effect on the ratio? Why?

3. Dividing the antecedent has what effect on the ratio? Why?

4. Dividing the consequent has what effect on the ratio? Why?

5. Multiplying both terms of a ratio by the same number has what effect on the ratio? Why?

6. Dividing both terms of a ratio by the same number has what effect on the ratio? Why?

SIMPLE PROPORTION.

275. A simple proportion is an equality between two ratios, and subsists between *four* terms, when the ratio of the first to the second equals the ratio of the third to the fourth. Thus, $3 : 12 = 14 : 56$, and is read, the ratio of 3 to 12 equals the ratio of 14 to 56. This equality of ratios is indicated by a double colon, thus, $3 : 12 :: 14 : 56$.

276. In every proportion the product of the extreme terms equals the product of the mean terms; that is, $3 \times 56 = 12 \times 14$.

277. This principle enables us to find any one of the four terms of a proportion, when the other three are known.

ILLUSTRATIONS.

1. *To find the fourth term, divide the product of the second and third terms by the first.*

2. *To find the third term, divide the product of the first and fourth terms by the second.*

3. To find the second term, divide the product of the first and fourth terms by the third.

4. To find the first term, divide the product of the second and third terms by the fourth.

Illustration.—The first, second, and third terms are 9, 18, and 5; what is the fourth term? This incomplete proportion is written, $9:18::5:$ what number? Since 5×18 , or 90, is 9 times the number, the fourth term must be $90 \div 9$, or 10. The complete proportion is $9:18::5:10$.

EXERCISES.

Find the missing term in the following:

1. $2:6::4:$ what number?
2. $7:14::10:$ what number?
3. $11:22::9:$ what number?
4. $2:6::—:12.$ 9. $11:—::9:18.$
5. $7:14::—:20.$ 10. $—:6::4:12.$
6. $11:22::—:18.$ 11. $—:14::10:20.$
7. $2:—::4:12.$ 12. $—:22::9:18.$
8. $7:—::10:20.$

Give a full analysis in each of the preceding examples, and show how the general principle in 276 applies.

278. As a ratio can subsist only between quantities of the same denomination, the missing term in any couplet must be of the same denomination as the other term of that couplet.

Illustration.—In the proportion 10 men : \$50 :: 30 men : \$150, the terms should stand, 10 men : 30 men :: \$50 : \$150. Hence, when the missing term is *men* the *third* term must be *men*; when the missing term is *dollars* the *third* term must be *dollars*.

279. The *third* term in every proportion is always

of the same denomination as the missing term, which is *always* the *fourth* term in the complete proportion.

280. In solving a problem by proportion, the first thing to be determined is the *denomination* of the fourth, or missing term. This determines the third term.

ORAL EXERCISES.

Determine the third term.

1. If 8 men can do a piece of work in 2 days, how many men will be required to do the same work in 4 days?

2. If \$30 purchase 5 barrels of flour, how many barrels will \$60 purchase?

3. If you walk 9 miles in 3 hours, how many miles can you walk in 18 hours?

4. If it take 3 hours to walk 9 miles, how many hours will it take to walk 27 miles?

281. The fourth term in every proportion must be *less* than, *equal to*, or *greater* than the third term.

ORAL EXERCISES.

Should the fourth term be greater, or less, than the third in the four preceding examples (280)? In the following?

1. Five hats cost \$20 ; will 40 hats cost more, or less? Why?

2. Ten horses cost \$1000 ; will 60 horses cost more, or less? Why?

3. Eight dollars purchase 2 hats ; will \$56 purchase more, or less? Why?

4. Twenty cents buy 5 oranges ; will \$2 buy more, or less? Why?

5. Twenty oranges cost \$1 ; will 3 oranges cost more, or less? Why?

6. A man can walk 20 miles in 5 hours ; can he walk a greater, or less, number of miles in 2 hours? Why?

7. Twelve men do a job of work in 6 days ; can 4 men do it in a greater, or less, time? Why?

282. If from the nature of the question the fourth, or missing, term should be *greater* than the third term, the *greater* of the two remaining terms must be the second, and the *less* of the two remaining terms must be the first term of the proportion.

1. *If 4 horses cost \$500, what will 20 horses cost?*

Solution. — As the answer is to be dollars, the third term of the proportion must be 500. Since 4 horses cost \$500, 20 horses will cost *more* than \$500 ; hence 20 is the second term of the proportion, and 4 is the first term. Thus, 4 : 20 :: \$500 : the missing term.

Solving this, we get $\frac{20 \times 500}{4} = 2500$. Hence, 20 horses will cost \$2500.

2. *If \$500 buy 4 horses, how many horses will \$2500 buy?*

Solution. — As the answer is to be horses, the third term must be 4. Since \$500 buy 4 horses, \$2500 will buy a *greater* number of horses ; hence 2500 is the second term of the proportion, and 500 is the first term. Thus, 500 : 2500 :: 4 : the missing term. Solving

this, we get $\frac{4 \times 2500}{500} = 20$. Hence, \$2500 will buy 20 horses.

283. If from the nature of the question the fourth, or missing, term should be *less* than the third term, the

less of the other two remaining terms must be the second, and the *greater* of the other two remaining terms must be the first term of the proportion.

1. If \$2500 buy 20 horses, how many horses will \$500 buy?

Solution. — As the answer is to be horses, the third term must be 20. Since \$2500 buy 20 horses, \$500 will buy a *less* number of horses; hence \$500 is the second, and \$2500 the first term of the proportion. Thus, 2500:500::20: the missing term. Solving this, we get $\frac{20 \times 500}{2500} = 4$. Hence, \$500 will buy 4 horses.

2. If 20 horses cost \$2500, what will 4 horses cost?

Solution. — As the answer is to be dollars, the third term must be dollars. Since 20 horses cost \$2500, 4 horses will cost *less*; hence 4 is the second, and 20 is the first term of the proportion. Thus: 20:4::2500: the missing term. Solving this, we get $\frac{4 \times 2500}{20} = 500$. Hence, 4 horses will cost \$500.

STATEMENT IN PROPORTION.

RULE. — I. *Determine the denomination of the answer.* (Art. 278.)

II. *The given term which is of the same name as the answer, must be made the third term of the proportion.*

III. *If from the nature of the question the answer should be greater than the third term, the greater of the two remaining terms must be made the second term of the proportion.* (Art. 282.)

IV. *If from the nature of the question the answer*

should be less than the third term, the less of the two remaining terms must be the second term of the proportion. (Art. 283.)

V. Multiply the third term by the second, and divide the product by the first term. (Art. 276.)

NOTE. — If the terms in any couplet contain different denominations, reduce them to the lowest denomination in either.

MISCELLANEOUS EXAMPLES.

1. If 4 oranges cost 24 cents, what will 9 oranges cost? 54 cents.

2. A man can walk 27 miles in 9 hours; how far can he walk in 5 hours?

3. The interest on \$315 for a given time at a given rate per cent is \$31.50; what is the interest on \$231 for the same time at the same rate?

4. If \$1624 yield \$16.24 interest in a certain time at a certain rate, what principal, in the same time at the same rate, will yield \$50.75 interest? \$5075.

5. A and B in trading farms value them respectively at \$50.15 and \$65 per acre. If the real value of B's is \$52 per acre, what is the real value of A's per acre?

6. If 92 yards of cambric line 75 yards of broad-cloth when the cambric is 3 quarters wide, how many yards will be required to line it when the cambric is $1\frac{1}{2}$ quarters wide?

7. A marble slab 20 feet long, 3 feet wide, 6 inches thick, weighs 1200 pounds. Find the length of a slab

of equal width and thickness, and of equal density, which weighs 2700 pounds.

8. Find the cost of $7\frac{1}{2}$ acres, if $9\frac{1}{4}$ cost \$148.

9. My salary, paid in currency, is \$2250 on a gold basis when gold is quoted at 113. What is my salary worth when gold is quoted $112\frac{1}{2}$?

10. If $6\frac{1}{2}$ yards of silk cost \$19 $\frac{1}{2}$, how much will $3\frac{1}{2}$ yards cost?

11. If $\frac{2}{3}$ of a barrel of flour cost \$3 $\frac{3}{8}$, what is the cost of $7\frac{1}{2}$ barrels.

12. Four men can do $\frac{4}{5}$ of a piece of work in $\frac{7}{8}$ of a week, in how many days can they do $\frac{2}{3}$ of it?

13. How many more revolutions will the fore-wheel of a carriage, which is 11 feet 9 inches in circumference, make, than the hind-wheel which is 13 feet 2 inches, in going one mile?

14. Find the value of $\frac{3}{8}$ of a business, $\frac{4}{7}$ of which is valued at \$3600.

15. At a certain time of day a pole 15 feet in length casts a shadow 20 feet long; find the height of a steeple which casts a shadow 136 feet at the same time of day.

16. At what time between 1 and 2 o'clock do the hands point in opposite directions?

17. At what time between 3 and 4 o'clock do the hands of a watch point in opposite directions?

18. At what time between 4 and 5 o'clock are the hands together?

19. When wheat is selling at 7s. 6d. per bushel, a baker's loaf weighs 9oz., how many ounces should it weigh when wheat sells for 6s. per bushel?

20. A farm containing 46 A. 134 P. rents for \$374.70 per annum, what will be the rent on 35 A. 90 P. at the same rate?

21. If 12 men can do a piece of work in $5\frac{1}{3}$ days, in how many days can 8 men and 5 boys do the work, if a man can work $2\frac{1}{2}$ times as fast as a boy?

22. A prisoner escaped from jail and traveled at the rate of 6 miles an hour. In $5\frac{1}{2}$ hours the constable started in pursuit, traveling at the rate of 9 miles an hour. In how many hours after breaking jail was the prisoner overtaken?

23. A and B can perform a piece of work in $5\frac{5}{11}$ days. A and C can do the same work in 6 days. B and C can do it in $6\frac{2}{3}$ days; in what time can each do the work alone, and how long will it take all of them together to do it?

COMPOUND PROPORTION.

284. A compound proportion involves two or more simple proportions; and any problem in compound proportion may be solved as a question in simple proportion, by making several successive statements.

ILLUSTRATION.

1. If 4 men earn \$20 in 5 days, how much can 7 men earn in 12 days?

Solution.—As the answer is to be dollars, the third term must be 20. Since 4 men earn \$20, 7 men can earn more. Hence, $4:7::20:35$. Again: Since \$35 are earned in 5 days, more can be earned in 12 days; hence, $5:12::35:84$. Hence, 7 men in 12 days earn \$84.

STATEMENT IN COMPOUND PROPORTION.

ARRANGEMENT OF CONDITIONS.

1st. Condition: 4 men, \$20, 5 days.

2d. Condition: 7 men, — 12 days.

Statement.—As the answer is to be dollars the third term must be 20. Since 4 men earn \$20, 7 men earn more. Then, the first couplet reads 4 : 7. Since \$20 are earned in 5 days, in 12 days more can be earned. Then, the second couplet reads 5 : 12. Then,

$$\left. \begin{array}{l} 4 : 7 \\ 5 : 12 \end{array} \right\} :: 20 : \text{the missing term. Solving this, we get } \frac{7 \times 12 \times 20}{4 \times 5} = 84.$$

2. If 20 men, by working 3 hours a day, can do a piece of work in 15 days, how many men will be required to do the same work in 12 days, working 5 hours a day?

ARRANGEMENT OF CONDITIONS.

1st. Condition: 20 men, 3 hours, 15 days.

2d. Condition: — men, 5 hours, 12 days.

1st. What is the third term? Why?

2d. Will it require more, or less, men to do the work in 12 days than it will to do it in 15 days? Why?

3d. Will it require more, or less, men to do the work when they work 5 hours a day, than it will when they work 3 hours a day? Why?

Statement.— $\left. \begin{array}{l} 12 : 15 \\ 5 : 3 \end{array} \right\} :: 20 : \text{the missing term.}$

Solving this, we get $\frac{3 \times 15 \times 20}{5 \times 12} = 15$ men for the answer.

STATEMENT IN COMPOUND PROPORTION.

RULE.—I. *Arrange the terms in the second condition under the corresponding terms in the first condition.*

II. *Select the third term as in simple proportion.*

III. *Arrange the terms in each couplet (without reference to any of the other couplets) as in simple proportion.*

IV. *Multiply the third term by the continued product of the second terms of the several couplets, and divide this product by the continued product of the first terms of the several couplets.*

NOTE. — If any couplet contains different denominations, reduce both terms to the lowest denomination in either term of the couplet.

MISCELLANEOUS EXAMPLES.

1. How many men will be required to do a piece of work in 25 days, working 7 hours each day, if 30 men can do it in 14 days, working 15 hours each day?

2. Four men by working 10 hours a day for 12 days can dig a ditch 24 yards long, 9 feet wide, 5 feet deep; how many hours a day must 3 men work to dig a similar ditch 10 yards long, 12 feet wide, 6 feet deep, in 10 days?

3. A loaned B \$1500 for 9 months when money was worth 7 per cent.; for what time must B loan A \$1800 when money is worth $3\frac{1}{2}$ per cent. per annum?

4. What sum of money loaned at 6 per cent. for 10 months will yield the same income as \$750 loaned for 12 months at 4 per cent.?

Ans. \$600.

5. It requires 200 men to build a stone wall 100 rods long, 4 feet high, $3\frac{1}{2}$ thick, in 2 days, working 6 hours a day. How many men will be required to build a

wall 75 rods long, 5 feet high, 7 feet thick, if the work is $2\frac{1}{2}$ times as difficult?

6. If 5 oxen or 7 cows eat the grass on $3\frac{4}{11}$ acres in 87 days, in what time will 3 cows and 2 oxen eat the grass on $10\frac{1}{11}$ acres.

7. A man bought city lots at \$40 per front foot, which were supposed to measure 961 feet. There was an error of $\frac{3}{8}$ of an inch in the 12 foot pole used in measuring it. How much was he cheated?

8. A marble slab 15 feet long, $2\frac{1}{2}$ feet wide, 4 inches thick, weighs 2000 lbs.; what is the thickness of a similar slab 25 feet long, 5 feet wide, weighing 3500 lbs?

9. It requires $\frac{1}{8}$ of a bushel of oats to keep 4 horses $\frac{1}{8}$ of a day, how many horses will 9 bushels feed $\frac{3}{8}$ of a day?

Ans. 160.

10. If 20 acres in St. Louis are worth 30 acres in Cincinnati, 50 acres in Cincinnati are worth 45 acres in Chicago, 25 acres in Chicago are worth 20 acres in Boston, and 33 acres in Boston are worth 40 acres in New Orleans, how many acres in St. Louis are equal in value to 1800 acres in New Orleans?

11. Forty men agree to do a piece of work in 50 days, but after working 9 hours per day for 30 days, they find they have completed but $\frac{1}{2}$ the work. How many extra men must they employ to finish the work according to agreement by working 10 hours per day?

12. Fifteen men eat 13 shillings' worth of bread in 6 days, when wheat is sold at 12 shillings per bushel; in what time will 30 men eat $43\frac{1}{8}$ shillings' worth, when wheat is selling at 10 shillings per bushel?

13. If $10\frac{1}{4}$ yards are required to make a suit of clothes, how many yards must a man buy when the cloth is $1\frac{7}{8}$ yards wide, so as to allow for shrinkage 5 per cent. in length, and the same in width?

14. If the type for a book of 84 pages, 50 lines on a page, lines averaging 8 words, $1\frac{1}{2}$ syllables to a word, $2\frac{1}{2}$ letters to a syllable, was set by two men in five days of 12 hours each, how many pages of a book, each page containing 75 lines, averaging $5\frac{1}{2}$ words each, 2 syllables to a word, 3 letters to a syllable, would be set by 7 men, laboring 8 hours a day for the working days of a week?

15. If 12 men, in two months of 4 weeks each, working 6 days per week, 12 hours a day, can set the type for 12 books of 600 pages, 120 lines to a page, 20 words to a line, 10 letters to a word, in how many months of $4\frac{1}{2}$ weeks each, will 7 boys, working 4 days per week, 16 hours a day, set the type for 6 books, of 500 pages each, 150 lines to the page, lines averaging 24 words, $4\frac{1}{2}$ letters to a word, each boy doing $\frac{5}{7}$ of the work of a man?

285. GENERAL REVIEW.

I. RATIO.

1. Define ratio.
2. Simple ratio.
3. Compound ratio.
4. Antecedent; consequent.
 - a. Which is dividend?
 - b. Which is divisor?

5. Define couplet.
6. How many terms in a couplet?
7. How is ratio indicated?
8. Is the ratio abstract or concrete?
9. Ratio subsists between what quantities?
10. In a compound ratio how many couplets?
11. How changed to simple ratio?
12. State fundamental principles.
 - a. Multiplying antecedent.
 - b. Multiplying consequent.
 - c. Dividing antecedent.
 - d. Dividing consequent.
 - e. Multiplying antecedent and consequent.
 - f. Dividing antecedent and consequent.
13. Ratio between fractions.
14. Ratio between compound numbers.

II. SIMPLE PROPORTION.

1. What is a simple proportion?
2. How many terms?
3. Which extremes? Which means?
4. How many couplets?
5. What must be known?
6. Unknown term, how found?
7. Third term, how determined?
8. Second term, how determined?
9. Give the Rule.
10. State substance of the *Note*.

III. COMPOUND PROPORTION.

1. What is a compound proportion?
2. How many couplets?
3. Arrangement of conditions.
4. Couplets, how arranged?
5. Fourth term, how found?
6. Differs how from simple proportion?
7. How solved by several statements?
8. Each couplet connected with third term.
9. Each couplet independent of the others.
10. Give the Rule.

PERCENTAGE.

286. Percentage embraces the calculations which are based on one hundred as the standard or unit of measure.

287. Per cent. is an abbreviation of *per centum*, which means *by the hundred*. Thus :

One per cent., written .01, means *one hundredth of any thing*.

Two per cent., written .02, means *two hundredths of any thing*.

Five per cent., written .05, means *five hundredths of any thing*.

Ten per cent., written .10, means *ten hundredths of any thing*.

Twenty per cent., written .20, means *twenty hundredths of any thing*.

Seventy-five per cent., written .75, means *seventy-five hundredths of any thing*.

One hundred per cent., written 1.00, means *the whole of any thing*.

288. Per cent. being used to denote the *rate* or number of *hundredths* to be taken, *percentage* is often used to denote the *result* thus obtained.

289. The per cent. sign, written %, is read *per cent.*

290. The number on which percentage is estimated is called the **base**.

291. The **amount** denotes the **base** increased by the **percentage**.

292. The **difference** denotes the **base** diminished by the **percentage**.

EXERCISES.

1. Express three per cent. decimally. Express 4 % decimally.

2. Express six per cent. decimally. Express 7 % decimally.

3. Express eight per cent. decimally. Express 9 % decimally.

4. Express eleven per cent. ; 12 % ; 13 % ; 14 % ; 15 %.

5. What part of any thing is 10 % of it?

6. What part of any thing is 25 % of it?

7. What part of any thing is 20 % of it?

8. What part of any thing is 50 % of it?

9. What part of any thing is $33\frac{1}{3}$ % of it?

10. What part of any thing is 40 % of it?
11. What part of any thing is 60 % of it?
12. What part of any thing is $66\frac{2}{3}$ % of it?
13. What part of any thing is $12\frac{1}{2}$ % of it?
14. What part of any thing is $16\frac{2}{3}$ % of it?
15. One-fifth of 100 is what % of 100?
16. One-tenth of 100 is what % of 100?
17. One-twentieth of 100 is what % of 100?
18. One-third of 100 is what % of 100?
19. One-fifth of any thing is what per cent. of that thing?
20. One-tenth of any thing is what per cent. of it?
21. One-twentieth of any thing is what per cent. of it?
22. One-third of any thing is what per cent. of it?
23. 50 % of any thing represents what part of it?
24. $33\frac{1}{3}$ % of any thing represents what part of it?
25. What per cent. expresses the whole of any thing?
26. Write from one per cent. to one hundred per cent. inclusive.
27. Write in the form of a common fraction, in its lowest terms, an equivalent for each of the above per cents.

EXAMPLES.

1. Find 2 % of 100 ; 1 % of 200.
2. Find 3 % of 100 ; 1 % of 300.
3. Find 4 % of 100 ; 1 % of 400.
4. Find 5 % of 100 ; 1 % of 500.
5. Find 4 % of 500 ; 5 % of 400.
6. Find 5 % of 600 ; 6 % of 500.

7. Find the difference between 8 % of 900 and 9 % of 800.

8. Find the difference between 12 % of 1000 and 7 % of 1000.

9. $33\frac{1}{3}$ % less 25 % of any thing is what % of it?

10. 25 % less $12\frac{1}{2}$ % of any thing is what % of it?

11. One-eighth of 64 less $12\frac{1}{2}$ % of 64 equals what?

12. One-sixth of 90 less $16\frac{2}{3}$ % of 90 equals what?

13. Find 5 % of 125 ; of 225 ; of 150 ; of 250 ; of 500 ; of 750.

14. Find 7 % of 80 ; of 800 ; of 1800 ; of 2800.

15. Find $12\frac{1}{2}$ % of 80 ; of 96 ; of 144 ; of 256 ; of 512.

16. Find 40 % of 400 ; 400 % of 40 ; 3 % of 30 ; 30 % of 300.

17. Find 10 % of 525 ; of 792 ; of 928 ; of 829 ; of 279.

18. What part of \$450 is $12\frac{1}{2}$ % of it?

19. What part of 1450 is $12\frac{1}{2}$ % of it?

293. Given the base and rate to find the percentage.

1. Find 12 % of 1875.

Suggestion.—12 per cent. equals how many *hundredths*?

.01 of 1875 equals what?

.12 of 1875 equals what?

2. What is 15 % of \$1875? Of \$7581?

3. What is 31 % of 1927? Of \$2719?

4. What is 27 % of 3264 sheep? Of \$3264?

5. What is $33\frac{1}{3}$ % of 939 horses? Of \$939?

294. Given the percentage and base to find the rate.

1. \$25 is what per cent. of \$125.

Suggestion. — \$1 is what part of \$125?

\$25 is what part \$125?

Find the decimal equivalent.

$\frac{1}{125}$.

$\frac{25}{125}$, or $\frac{1}{5}$.

.20, or 20 %.

2. \$8 is what per cent. of \$40? Of 48? Of 64?

3. 9 is what per cent. of 27? Of 36? Of 45?

4. 6 is what per cent. of 150? Of 200? Of \$600?

5. A man gained \$12 on \$200; what was the rate of gain?

295. Given the percentage and rate to find the base.

1. A man paid \$12 for the use of a certain amount of money, at the rate of 6 %; what amount did he borrow?

Suggestion. — How many cents did he pay on each dollar?

6 cents is what part of 100 cents?

Then, \$12 is what part of the amount borrowed?

Finally, \$12 is $\frac{6}{100}$ of what number?

2. What sum can I borrow for \$18, at 9 %?

3. What sum will yield \$25, at 5 %?

4. If \$40 is 10 % of a man's money, how much has he?

5. If \$81 is 9 % of a man's money, how much money has he?

6. How many oranges can you buy for 81 cents, if you pay 3 cents for one orange?

7. How many dollars can you borrow for 81 cents, if the use of \$1 is worth 3 cents?

NOTE.—The principles involved in Articles 293, 294, 295, are *fundamental*, and underlie the entire subject of percentage, and its applications. They may be stated as follows:

First Principle.

The percentage is the same part of the base that the rate is of 100.

Second Principle.

The rate is the same part of 100 that the percentage is of the base.

Third Principle.

The percentage is the same part of the rate that the base is of 100.

296. The above principles, though differently worded, are substantially the same.

Any two of the elements, *rate*, *percentage*, *base*, being given, the third element may be found by applying some one of these principles.

NOTE.—Let the pupils write a rule for solving examples arising under 294 and 295.

MISCELLANEOUS EXERCISES.

1. 40 is 10 % of what number?
2. 40 is one-tenth of what number?
3. $16\frac{2}{3}$ is what part of 100?
4. 25 is $16\frac{2}{3}$ % of what number?
5. 1 is what per cent of 6? Of 8? Of 10?
6. 32 is what per cent of 64? Of 96? Of 16?
7. 27 is what per cent of 81? Of 105? Of 54?
8. What per cent of 24 equals 8?

9. What per cent of 91 equals 13?
10. Find 7 % of 216. Of \$216. Of 216 sheep.
Of 216 acres.
11. 51 is $33\frac{1}{3}$ % of what number?
12. 48 is $66\frac{2}{3}$ % of what number?
13. 36 is $16\frac{2}{3}$ % of what number?
14. 64 is $6\frac{1}{4}$ % of what number?
15. What per cent of 72 is 36? 24? 48? 12?
16. What per cent of 21 is 7? 14? 3? 21?
17. What per cent of 400 is 5? 6? $8\frac{1}{8}$? $12\frac{1}{2}$? 20?
25? $33\frac{1}{3}$?
18. Is 20 a greater, or less, per cent. of 200 than it
is of 300? Why?

SIMPLE INTEREST.

- 297.** Interest is money paid for the use of money.
- 298.** Interest is *simple*, *annual*, or *compound*.
- 299.** Simple Interest is interest on the principal for
a given time at a given rate.
- 300.** The **principal** is the money on which interest
is paid.
- 301.** The **rate** is the *per cent.*, *per annum*.
- 302.** The **amount** is the sum of principal and interest.
- 303.** **Legal interest** is the rate established by law.
- 304.** **Usury** is interest above the legal rate.
- 305.** **Compound interest** is interest on interest.
- 306.** Every problem, in simple interest, involves five
quantities.
- a. *The Principal.*
- b. *The Rate.*

- c. *The Time.*
- d. *The Interest.*
- e. *The Amount.*

307. Any three of these five elements being given, the other two may be found. Hence,

308. There are twenty possible problems in interest.

CASE I.

309. Given the principal, rate, and time to find the interest.

1. What is the interest on \$225, for 2 years, 6 months, at 6 %?

Suggestion. — What is the interest on \$225 for 1 year, at 6 per cent?

What is the interest on \$225 for 2 years, at 6 per cent?

What is the interest on \$225 for $\frac{1}{2}$ a year at 6 per cent?

What is the interest on \$225 for $2\frac{1}{2}$ years at 6 per cent?

NOTE. — this example the interest for one year is 6 per cent., of \$225.

The interest for two years is twice the interest for one year.

The interest for 6 months is one-half the interest for one year.

2. Find the interest on \$500 for 4 years, 6 months, at 6 %.

3. Find the interest on \$1000 for 6 years, 4 months, at 6 %.

4. Find the interest on \$2500 for 8 years, 3 months, at 6 %.

5. Find the interest on \$800 for 3 years, 3 months, and 10 days, at 6 %.

Suggestion.—6 per cent. of \$800 is the interest for 1 year.

What is the interest for 3 years?

10 days is what part of a month?

$3\frac{1}{2}$ months is what part of a year?

What is the interest for $3\frac{1}{2}$ months?

6. What is the interest on \$100 for 4 years, 4 months, and 15 days, at 6 %?

7. Find the interest on \$200 for 5 years, 9 months, and 20 days, at 6 %.

8. Find the interest on \$300 for 6 years, 2 months, 24 days, at 6 %.

9. Find the interest on \$400 for 7 years, 10 months, 6 days, at 6 %.

10. Find the interest on \$500 for 8 years, 8 months, 18 days, at 6 %.

SIX PER CENT. METHOD.

310. Very generally in this country the year is regarded as consisting of 12 months of 30 days each.

311. When this is the case the following method of computing interest is both convenient and expeditious.

ANALYSIS OF CONDITIONS.

Since the interest on \$1 is 6 cents for 12 months; then,

- a. The interest on \$1 is 1 cent for every 2 months.
- b. The interest on \$1 is 5 mills for 1 month or 30 days.
- c. The interest on \$1 is 1 mill for every 6 days.
- d. The interest on \$1 is $\frac{1}{3}$ of a mill for 1 day. Hence,

312. To compute interest at six per cent., for any number of years, months, and days.

I. Reduce the given number of years to months, and add the given number of months.

II. *Take one-half this number of months for the interest on \$1, expressed in cents.*

III. *Take one-sixth of the given number of days for the interest on \$1, expressed in mills.*

IV. *Take one-sixth of a mill for each remaining day.*

V. *Multiply this interest on \$1 for the whole time by the given number of dollars. The product will be the interest required.*

EXAMPLES.

1. What is the interest on \$1, at 6 %, for 2 months? 4 months? 6 months? 8 months? 10 months? 14 months? 16 months? 18 months? 20 months? 22 months?

2. What is the interest on \$2 for 8 months? \$3 for 10 months?

3. Find the interest, at 6 %, on \$12 for 24 months? \$24 for 12 months?

4. Find the interest, at 6 %, on \$16 for 18 months? \$18 for 16 months.

5. Find the interest, at 6 %, on \$216 for 40 months.

6. Find the interest, at 6 %, on \$1 for 12 days; 14 days; 16 days.

7. Find the interest, at 6 %, on \$1 for 18 days; 24 days; 48 days.

8. Find the interest, at 6 %, on \$12 for 12 days; 60 days; 90 days.

9. Find the interest, at 6 %, on \$124 for 24 days; 64 days; 84 days.

10. Find the interest, at 6 %, on \$124 for 90 days ; 60 days ; 30 days.

Find the interest, at 6 %.

11. On \$1 for 8 months and 18 days.

12. On \$1 for 10 months and 24 days.

13. On \$15 for 24 months and 24 days.

14. On \$150 for 24 months and 24 days.

15. On \$1500 for 240 months and 240 days.

16. On \$250 for 1 year, 6 months and 12 days.

17. On \$1250 for 2 years, 8 months and 18 days.

18. On \$2727 for 4 years, 10 months and 24 days.

19. On \$7272 for 10 years, 4 months and 6 days.

20. On \$25116 for 5 years, 2 months and 12 days.

21. On \$1 for 1 year ; for 1 month ; for 1 day.

22. On \$5 for 5 years ; for 5 months ; for 5 days.

23. On \$1 for 1 year, 1 month and 1 day.

24. On \$5 for 5 years, 5 months and 5 days.

25. On \$125 for 8 years, 9 months and 10 days.

313. To compute interest at any rate per cent., for any time.

RULE I. *Compute the interest at six per cent., for the given time. (Art. 212.)*

II. *Take such part of this interest as the given rate is of six per cent.*

1. Find the interest, at 3 %, on \$20 for 1 year, 8 months, 18 days.

Suggestion. — Find the interest, at 6 per cent., for the given time.

3 is what part of 6?

Then the interest, at 3 per cent., is what part of the interest at 6 per cent?

2. Find the interest on \$72 for 2 years, 9 months, 13 days, at 4 %.

Suggestion. — What is the interest on \$72 for the given time, at 6 per cent.?

4 is what part of 6?

Then the interest, at 4 per cent., is what part of the interest at 6 per cent.?

3. Find the interest on \$321 for 3 years, 11 months, 15 days, at 7 %.

Suggestion. — What is the interest at 6 per cent.?

7 is what part of 6?

Then the interest, at 7 per cent., is what part of the interest at 6 per cent.?

4. Find the interest on \$218 for 4 years, 7 months, 9 days, at $5\frac{1}{2}$ %.

Suggestion. — What is the interest at 6 per cent.?

$5\frac{1}{2}$ is $\frac{11}{12}$ of 6.

Then, what is the interest at $5\frac{1}{2}$ per cent.

In a similar manner explain the following examples :
Compute the interest

1. On \$100 for 6 months, 18 days at 2 %.
2. On \$125 for 9 months, 24 days, at 3 %.
3. On \$150 for 10 months, 21 days, at $3\frac{1}{2}$ %.
4. On \$175 for 11 months, 11 days, at 4 %.
5. On \$200 for 13 months, 12 days, at $4\frac{1}{2}$ %.
6. On \$225 for 1 year, 1 month, 1 day, at 5 %.
7. On \$250 for 2 years, 2 months, 2 days, at $5\frac{1}{2}$ %.
8. On \$275 for 3 years, 3 months, 3 days, at $5\frac{3}{4}$ %.
9. On \$300 for 4 years, 4 months, 4 days, at $6\frac{1}{2}$ %.
10. On \$325 for 3 years, 4 months, 5 days, at 7 %.
11. On \$350 for 4 years, 5 months, 6 days, at $7\frac{1}{2}$ %.
12. On \$375 for 5 years, 6 months, 7 days, at 8 %.

13. On \$400 for 6 years, 7 months, 8 days, at 9 %.
14. On \$4000 for 7 years, 8 months, 9 days, at 10 %.
15. On \$4375 for 8 years, 9 months, 10 days, at 12 %.
16. On \$3457 for 9 years, 8 months, 7 days, at 15 %.
17. On \$4735 for 10 years, 10 months, 10 days, at 10 %.
18. On $\$ \frac{1}{2}$ for $\frac{1}{2}$ of a year, $\frac{1}{2}$ of a month, $\frac{1}{2}$ of a day, at $\frac{1}{2}$ %.
19. On $\$ \frac{3}{4}$ for $\frac{3}{4}$ of a year, $\frac{3}{4}$ of a month, $\frac{3}{4}$ of a day, at $\frac{3}{4}$ %.
20. On \$1000 for 100 years, 10 months, 1 day, at 20 %.

NOTE.—A part of a day is either regarded as a whole day, or rejected altogether.

SECOND METHOD.

314. When the year is regarded as consisting of 360 days, the following method of computing interest is sometimes used :

ANALYSIS OF CONDITIONS.

- a. The interest on any principal at 6 %, for 1 year, is .06 of the principal.
- b. The interest on any principal for 1 month, is $\frac{1}{12}$ of the interest for 1 year.
- c. The interest for 1 day is $\frac{1}{360}$ of the interest for 1 month. Hence,

315. To compute the interest for years, months, and days.

RULE. — I. *To find the interest for one year multiply the given principal by the rate per cent.*

II. *To find the interest for one month take $\frac{1}{12}$ of the interest for one year.*

III. *To find the interest for any number of months multiply the interest for one month by the number of months.*

IV. *To find the interest for one day, take $\frac{1}{360}$ of the interest for one month.*

V. *To find the interest for any number of days multiply the interest for one day by the number of days.*

VI. *The sum of these several results is the interest required.*

1. Find the interest on \$72 for 3 years, 7 months, 18 days, at 6 %.

Suggestion. — What is the interest on \$72 for 1 year at 6 per cent?

What is the interest for 3 years at 6 per cent?

What is the interest for 1 month?

What is the interest for 7 months.

What is the interest for one day? For 18 days?

What is the interest for 3 years, 7 months, 18 days.

NOTE. — For exercises under this method let the pupils work the examples under. (Art. 313.)

EXACT METHOD.

316. When the year is regarded as consisting of 365 days the following method may be employed:

RULE. — I. *Express the given time in days.*

II. *To find the interest for one year multiply the principal by the rate per cent.*

III. *Multiply the interest for one year by the given number of days and divide the product by 365.*

1. Compute the exact interest on \$2112 for 120 days, at 8 %.
2. Compute the exact interest on \$1640 for 63 days, at 7 %.
3. Compute the exact interest on \$1880 for 93 days, at 9 %.
4. Compute the exact interest on \$1492 for 76 days, at 10 %.
5. Compute the exact interest on \$1688 for 27 days, at 12 %.

Relations subsisting between **Principal, Rate, Time, Interest, Amount.**

317. Any three of these five elements being given, the others can be computed.

318. Under 309, Case I., one of these relations has already been considered.

CASE II.

319. Given the principal, time and interest to find the rate.

1. The interest on \$1200 at a certain rate, for 2 years, 6 months, is \$150. Find the rate per cent.

Suggestion. — What is the interest on \$1500 for the time, at 1 per cent? Since the interest is \$30 at 1 %, for the given time, what rate will yield \$150 in the same time?

NOTE. — In this case \$1200 earns, in the given time, \$150, at the required rate. But \$1200, in the same time, earns \$30 at 1 per cent. Then, at what rate will \$1200 earn \$150?

The following is a similar question :

2. At what *wages per day* can a certain number of men earn \$150 in a certain time, if, in the same time, the same men can earn \$30, at \$1 *per day*?

3. At what *rate per annum* must \$400 be put at interest for 10 years to yield \$100. Ans. $2\frac{1}{2}$.

4. If \$2700 on interest for 2 years and 8 months, yields \$378, what is the rate per cent?

5. At what rate will *any* principal double itself in 20 years?

Suggestion. — At what rate will it double itself in 1 year?

At what rate will it double itself in 20 years?

CASE III.

320. Given the rate, interest and time to find the the principal.

1. What principal at 6 % will yield \$206.25 interest in 2 years and 9 months?

Suggestion. — What is the interest on \$1 for the given time, at 6 per cent?

If \$1 yields \$.165 in the same time, at the same rate,

How many dollars will yield \$206.25?

2. What principal will gain \$200 in 4 years, 6 mos., at 6 %?

3. What principal will gain \$384 in 4 years, 3 months, 6 days, at 6 %?

4. At 8 % what principal will gain \$8651.20 in 10 yrs., 3 mos., 15 days?

5. At 7 % what principal will gain \$750 in 3 yrs., 5 mos., and 10 days?

6. Find the amount in each of the above examples.

CASE IV.

321. Given the principal, rate and interest to find the time.

1. In what time will \$560 gain \$168, at 6 % per annum?

Suggestion. — What is the gain on \$560 in 1 yr., at 6 per cent.?
Then, in what time will \$560 gain \$168?

2. In what time will \$250 gain \$28.125, at 6 % per annum?

3. In what time will \$240.80 gain \$84.28, at 6 % per annum?

4. In what time will \$196 gain \$98.49, at 9 % per annum?

5. In what time will any principal double itself, at 6 %?

Suggestion. — In what time will any principal double itself at 1 per cent.?

Then, in what time will it double itself at 6 per cent.

6. In what time will any principal treble itself at 5 %?

Suggestion. — The whole gain equals what part of the principal?

In what time, at 1 per cent., will the interest equal *once* the principal?

In what time, at 1 per cent., will the interest equal *twice* the principal?

In what time, at 5 per cent., will the interest equal twice the principal?

CASE V.

322. Given the amount, rate and time to find the principal.

1. Given the amount \$287.525, rate 10 %, time 5 yrs., 6 mos. Find the principal.

Suggestion. — What is the amount of \$1 for the given time, at 10 per cent.?

Then, what principal will amount to \$287.525?

2. What principal will amount to \$804 in 4 yrs., 4 mos., at 5 %?

3. What principal will amount to \$408 in 3 yrs., 3 mos., at 6 %?

4. What principal will amount to \$84 in 1 yr., 9 mos., 18 days, at 7 %?

5. What principal will amount to \$48 in 2 yrs., 8 mos., 12 days, at 12 %?

PROMISSORY NOTES.

323. A Promissory Note is a promise, in writing, to pay to some particular person a specified sum of money, at a specified future time.

324. Every valid promissory note contains

- a. A distinct promise to pay.
- b. A specified sum of money.
- c. A specified time.
- d. The name of the payee.
- e. The date of the note.
- f. The name of the maker of the note.

325. The words **value received** are generally inserted in the body of a note, but are not necessary to give it validity.

326. **Interest** may, or may not, be specified in the note. If not specified, the legal rate of the State where the note is payable is understood.

327. A note may be made payable on demand, or in a specified time after date.

328. A **note on demand** draws legal interest, when none is specified, only **after demand** has been made.

329. A **note on time** draws legal interest, when none is specified, only **after maturity**. If specified, then from date.

330. The maturity of a note is the time when it becomes due, which is generally **three days after** the time specified in the note.

331. These three extra days are known as **days of grace**.

332. When the last day of grace falls on a national holiday, Fourth of July, Christmas, New Years, Sunday, only **two** extra days are allowed.

333. Promissory notes are **Negotiable**, or **Non-Negotiable**.

334. A **Negotiable** note is payable to some specified person or his order. The words **or order** follow the name of the payee.

335. A **Non-Negotiable** note does not contain the words, **or order**.

336. A **Negotiable** note is transferrable only by endorsement.

337. Only the payee, or holder of a note, can endorse it for the purpose of transferring title; which consists in writing his name across the back of it.

338. An endorsement may be in **blank, in full, or without recourse.**

339. A person not a party to a note may endorse it by writing his name across its back. Under certain restrictions such endorsement makes the endorser responsible for the payment of the note, in case the maker fails to pay.

340. To make a party, foreign to a note, responsible as endorser, the endorsement must be made **before maturity.**

NOTE.—For other points, see Discounts.

Note on Demand.

\$800.

ST. LOUIS, MO., July 1, 1880.

On demand I promise to pay to C. K. Coon, or order, one hundred dollars. Value received. Interest at the rate of 8 per cent. per annum.

C. C. KOON.

Note on Time.

\$800.

ST. LOUIS, MO., July 1, 1880.

For value received I promise to pay Jack E. Georgeson, or order, one hundred and fifty dollars, thirty days after date, with interest at the rate of 7 per annum.

GUS E. JACKSON.

NOTE.—Had interest not been specified this note would not draw interest till after the expiration of the time specified.

Note Payable at Bank.

\$1000 $\frac{10}{100}$.

ST. LOUIS, MO., July 30, 1880.

Thirty days after date, for value received, I promise to pay Elias Hicks, or order, one thousand and $\frac{10}{100}$ dollars, with interest, after maturity, at the rate of ten per cent. per annum. Payable at the Bank of Commerce.

MOSES TOIL.

341. A bank check, payable to order, or bearer, is treated in many respects as a promissory note.

342. Bank bills are promissory notes payable on demand, and are transferrable by mere delivery.

BANK DISCOUNT.

343. Bank Discount is the interest for the time specified in a note, including the days of grace.

344. The Proceeds is the difference between the face of a note and the bank discount.

NOTE. — When a note is discounted sometime *after* its date, the *amount* of the note at the time of discount, is regarded as the face of the note.

345. To find the bank discount and net proceeds of a note.

1. Find the discount and the proceeds of a note for \$400, discounted at bank, for 60 days, the rate of discount being 6 %.

Suggestion. — What is the interest on \$400 for 63 days, at 6 %.
Then, what is the proceeds?

2. On \$100 discounted at bank for 60 days, at 6 %.

3. On \$150 discounted at bank for 30 days, at 6 %.

4. On \$175 discounted at bank for 30 days, at 7 %.

5. On \$250 discounted at bank for 90 days, at 7 %.

6. On \$350 discounted at bank for 90 days, at 8 %.

7. What sum of money would a bank pay on a note for \$1000 discounted for 90 days, at 9 %?

8. How much must a man pay a bank for the use of \$2512 for 60 days, their rate of discount being 10 %?

9. A man held a note payable six months after date, which he got discounted at bank four months after date. The note being drawn for \$5000, and the rate of discount being 8 %, how much did the bank pay him?

10. A note drawn for \$1520, payable five months after date, with interest at the rate of 7 %, was discounted at a bank two months after date, at the rate of 6 %. What was received on it?

Suggestion. — What was the amount, or face, of the note at time of maturity?

What the discount on this amount for balance of the time?

What the proceeds?

11. If the note in example 10 had drawn interest at the rate of 6 % per annum, and had been discounted at the rate of 7 % per annum, what would have been the net proceeds?

346. Given the time and rate of discount and the net proceeds to find the face of the note.

1. For what sum must a note be drawn that, when discounted at a bank for 90 days, at 6 %, the net proceeds will be \$1000?

Suggestion. — What is the discount on \$1 for 93 days, at 6 %? What the proceeds of \$1?

Then, to yield \$1000 proceeds, what sum is required?

Fuller Analysis.

The discount on \$1 for 93 days, at 6 %, is \$0.0155.

The proceeds of \$1 for 93 days, at 6 %, is \$0.9845.

It requires \$1 to yield \$0.9845 net proceeds.

How many dollars are required to yield \$1000 net proceeds?

For what sum must a note be given at bank

1. To realize \$144, the term of discount being 30 days, and the rate 6 %?

2. To realize \$200, the term of discount being 30 days, and the rate 6 %?

3. To realize \$300, the term of discount being 30 days, and the rate 6 %?

4. To realize \$400, the term of discount being 30 days, and the rate 7 %?

5. To realize \$500, the term of discount being 60 days, and the rate 7 %?

6. To realize \$600, the term of discount being 60 days, and the rate 7 %?

7. To realize \$750, the term of discount being 60 days, and the rate $7\frac{1}{2}$ %?

8. To realize \$800, the term of discount being 90 days, and the rate 8 %?

9. To realize \$850 the term of discount being 90 days, and the rate $8\frac{1}{2}$ %?

10. To realize \$900, the term of discount being 90 days, and the rate 9 %?

11. A man wishing to borrow \$1250 from a bank, for 90 days, at 10 per cent., computed the interest on \$12500 for 93 days, at 10 per cent., and gave his note for the amount. Did he cheat himself or the bank? How much?

12. Had he wished to borrow \$50000, on the same terms, how much more, or less, would he have to pay

at the maturity of the note, than he would have had to pay, if he had given his note for the correct amount?

NOTE. — Let the pupils make a rule for this and similar cases. Write them out carefully, with a view to brevity, clearness, and accuracy.

PROFIT AND LOSS.

347. In **Profit and Loss** four distinct elements are considered :

- a. The purchase price, or original number.
- b. The selling price, or resulting number.
- c. The gain, or loss.
- d. The rate of gain, or loss.

348. Given the purchase price and the rate to find the gain or loss.

1. Bought a horse for \$150, and sold it at a profit of 20 %. What was my gain?

Suggestion. — 20 per cent. of \$150 equals what?

Then, what was my gain?

And what was the selling price?

2. Bought a watch for \$75, and sold it at a loss of 25 %. What was the loss?

Suggestion. — 25 % of \$75 equals what?

Then, what was the loss?

And what was the selling price?

3. A man sold a house which cost him \$2000, for \$1500. What was his loss?

4. A farm, which was worth \$4560, sold at 15 % less than its value. What was the selling price?

5. On a house and lot which cost me \$7100, I lost

30 % by depreciation in value. What was it then worth?

6. A grocer sold sugar, which cost 9 cents a pound, at a profit of $33\frac{1}{3}$ %. What was his gain on 100 pounds?

7. A lot of damaged tea that cost 60 cents a pound, was sold at a loss of 32. %. The lot consisting of 40 chests, each weighing 50 pounds. What was the entire loss?

349. Given the purchase price and the selling price to find the rate of gain or loss.

1. Sold butter which cost 12 cents, at 15 cents a pound. What was the rate of profit?

Suggestion. — What was the gain on each pound?

The gain was what part of the cost of each pound?

The gain was what per cent. of the purchase price?

2. If eggs which cost 18 cents, sold for 15 cents a dozen, what was the loss per cent.?

Suggestion. — What was the loss on each dozen?

What part was the loss, of the purchase price?

Then, the loss was what per cent. of the purchase price?

NOTE. — The pupil must remember that the *actual* gain or loss, as well as the *rate* of gain or loss, is estimated on the purchase price.

350. Given the selling price and the rate to find the purchase price.

1. A man sold calicoes at 15 cents a yard, thereby gaining 25 %. What did they cost him?

Suggestion. — 25 % of the purchase price is what *part* of the purchase price?

Then, the selling price is how many *fourths* of the purchase price?

Then, 15 cents is $\frac{3}{4}$ of what number?

2. A merchant lost 10 % by selling silks at 90 cents a yard. What did they cost him?

Suggestion. — 10 % of the cost is what part of it?

For how many tenths of the cost did each yard sell?

Then, 90 cents is $\frac{9}{10}$ of what number?

3. If by selling goods at 48 cents a yard a man loses $33\frac{1}{8}$ %, what did they cost?

4. A house sold for \$1100, yielded a profit of 14 %. What was the cost of the house?

5. A man gained 12 % by selling his house and lot for \$2688. What did it cost him?

351. Given the gain and rate of gain to find the cost.

1. A farmer sold a team of horses at a gain of \$27 ; which was a profit of $8\frac{1}{8}$ %. What did the team cost him?

Suggestion. — $8\frac{1}{8}$ % is what part of 100 % ? That is,

The gain is what *part* of the cost?

Then, \$27 is $\frac{1}{12}$ of what number?

RULE. — *Divide the gain, or loss, by the rate of gain, or loss, expressed decimally. The quotient will be the cost.*

NOTE. — Let the pupils write rules for the other cases in Profit and Loss.

352. To find what goods must be marked to yield a given net profit after allowing a given discount.

1. What must goods be marked, that cost 20 cents a yard, to yield a net profit of 10 %, after allowing a discount of $8\frac{1}{8}$ % to a customer?

Suggestion.—For what must goods sell, that cost 20 cents, to net 10 % ?

$8\frac{1}{8}$ % discount is what part of 100 % ?

Then, the *discount* is what part of the marking price?

Then, the *selling price* is what part of the *marking price*?

Or, 22 cents is $\frac{1}{2}$ of what number?

2. Goods that cost 40 cents a yard sell at a net profit of 25 % after allowing a customer a discount of $16\frac{2}{3}$ %. What were they marked? 60 cents.

3. A merchant bought a lot of knives at $37\frac{1}{2}$ cents apiece. He wished to make a discount of $12\frac{1}{2}$ %, and a net profit of 10 %. Find the marking price.

4. At what price must an article which cost 30 cents be marked, to allow a discount of $12\frac{1}{2}$ %, and yield a net profit of $16\frac{2}{3}$ %?

5. At what price must land that cost \$100 per acre be held, that the owner may fall 30 % on the asking price, and realize a net profit of 20 %? \$180.

6. A man paid \$150 for a horse which he offered in trade at a price he was willing to discount at 40 % for cash, as he would then gain 20 %. What was his trading price? \$300.

7. A man bought steel-pens at a discount of 40 % on the retail price, and sold them at a profit of $33\frac{1}{3}$ %

% on his investment. If his net gain was \$27, what was the retail price? \$135.

Suggestion. — The gain is what part of the purchase price?

Then, what is the purchase price?

The purchase price is what part of the retail price?

Then, what is the retail price?

8. Find the retail price of an article bought at a discount of $33\frac{1}{8}\%$, which sold at an advance of $12\frac{1}{2}\%$, yields a net profit of \$8. \$96.

COMMISSION.

353. Commission is a certain percentage allowed an agent for transacting business for another person.

354. The commission of an agent is estimated on the amount invested, or on the amount received for his principal.

355. In problems of this class four elements are considered :

- a. The amount invested, or received by the agent.
- b. The rate of commission.
- c. The commission
- d. The net proceeds, or cost.

356. Hence, there are six classes of problems under this subject.

357. Given the amount invested, or received, and the rate to find the commission.

1. A commission merchant bought 1000 barrels of flour, at \$7.50, receiving a commission of $2\frac{1}{2}\%$ on the purchase. Find his commission. \$187.50.

Suggestion. — This is similar to Art. 348.

2. A real estate agent collected rents to the amount of \$2700. What was his commission at $2\frac{1}{2}\%$? What did he pay his employer?

358. Given the amount invested, or received, and the commission, to find the rate of commission.

1. An agent purchased wheat to the amount of \$12520 for a miller. His commission being \$187.80, what was his rate? $1\frac{1}{2}\%$.

Suggestion. — This is similar to ART. 349.

2. If the commission on an investment of \$27560, was \$206.70, what was the rate of commission?

359. Given the commission and rate to find the amount invested, or received.

1. If an agent's commission was \$125, and his rate of commission $2\frac{1}{2}\%$, what amount did he invest, or receive for his employer? \$5000.

Suggestion. — This is similar to ART. 351.

2. An agent's commissions during one year were \$2140; what were his collections, if his rate was $1\frac{3}{4}\%$.

360. Given the gross investment and the rate to find the net investment.

1. A real estate agent collected during the year \$81590. His rate being $2\frac{1}{2}\%$, what amount did he pay over to his principal?

Suggestion. — This is similar to ART. 350.

2. A merchant in St. Louis sent \$8523 to an agent in N. Y. to be invested in flour at \$5 per barrel. How

many barrels did he buy, and what was the agent's commission, his rate being $2\frac{1}{2}\%$?

BROKERAGE.

361. Brokerage is a percentage paid an agent for transacting business for another person, generally in *money, bonds, and stocks*.

362. Problems similar to those under Commission arise under Brokerage, and are solved by the application of the same principles.

363. These principles are fully discussed under ART. 347 to 351.

INSURANCE.

364. Insurance is guarantee given against loss.

365. A Policy of Insurance is a written agreement to indemnify a person in case of loss.

366. In problems of this class there are three elements to be considered :

- a. The amount insured.
- b. The rate of insurance.
- c. The premium.

367. Given the amount insured and the rate to find the premium.

1. What will it cost to insure a house valued at \$3000, the rate of insurance being $\frac{3}{4}\%$?

Suggestion. — This is similar to ART. 348.

2. The face of a policy of insurance on a store and

its contents, was \$12000, and the rate of insurance 1 %. Find the premium.

368. Given the amount of insurance and the premium, to find the rate of insurance.

1. The premium on a store and its contents, valued at \$96000, was \$1440. Find the rate of insurance.

2. When \$750 was paid to insure \$200000 worth of property, what was the rate?

3. Three-fifths of the value of the contents of a store, and $\frac{2}{3}$ of the value of the building were insured at a cost of \$500. The building being valued at \$10000, and the goods at \$60000, what was the rate of insurance?

369. Given the premium and the rate to find the face of the policy.

1. An insurance was effected on $\frac{3}{4}$ of the value of a store and contents, at $\frac{1}{2}$ %, by paying \$75. What was the face of the policy? What was the value of the property?

Suggestion. — This problem is similar to those under Art. 352.

2. Find the face of a policy of insurance when the premium paid was \$125, and the rate $\frac{7}{8}$ %.

3. A house insured for $\frac{2}{3}$ of its value was consumed by fire. The cost of the policy, at $\frac{3}{4}$ %, was \$140.75. What was the owner's loss? Ans. 9524.08 $\frac{1}{4}$.

370. To find the face of a policy to cover both insurance and premium.

1. What must be the face of a policy which, in case of fire, will completely indemnify the owner who pays a premium of \$145.25, when the rate is $\frac{1}{3}\%$?

Suggestion. — What sum will cover the property? Art. 369.

What must he receive to cover both property and premium?

This amount is the present worth of what sum discounted at $\frac{1}{3}\%$?

COMPOUND INTEREST.

371. Compound Interest is interest on both principal and interest, when the interest due is not paid.

372. Interest may be compounded annually, semi-annually, or quarterly.

1. What is the compound interest on \$1000 for 2 years, at 6 %

Explanation. — Find the amount of the principal for 1 year. Find the amount of this new principal and subtract \$1000....

OPERATION.

\$1000 Principal.

.06 Rate per cent.

Interest for the first year equals..... \$60.00

First principal \$1000.00

New principal for second year..... \$1060.00

.06 Rate per cent.

Interest for the second year equals..... \$63.60

The second principal..... \$1060.

Amount at the end of second year..... \$1123.60

Deduct first principal..... \$1000.00

Compound interest..... \$123.60

The interest for the first period is the simple interest on the principal for that period. The *amount* for the first period is the *principal*

for the second period, thus giving interest on both principal and interest for the second year.

The difference between the amount at the end of the last period, and the first principal, must show the gain, or compound interest.

373. Interest compounded semi-annually for 2 years, 6 months, gives *five* periods.

Find the compound interest

2. On \$500 due in 2 years ; in 3 years ; in 4 years.

3. On \$800 due in 18 months, payable semi-annually ; in 24 months.

4. On \$1200, payable quarterly, for 2 years.

5. On \$1 for 10 years, at 10 %. On \$1.50. On \$2.50.

NOTE.—Tables, prepared for that purpose, are generally used in solving examples involving compound interest.

DISCOUNT AND PRESENT WORTH.

374. Bank discount has already been considered. It corresponds to the simple interest on the principal, for the time, at the given rate.

375. True Discount differs from bank discount. It is the difference between the **Present Worth** of a debt and the **Face of the Debt**.

376. The **Present Worth** of a debt is the sum, which, placed at interest for the given time and at the given rate, will amount to the debt.

377. Given the principal, or debt, the time and rate to find the present worth.

1. What is the present worth of \$161.35, due in 1

year, 9 months, 24 days, at 6 %? What is the discount?

Suggestion.—What principal will, in 1 year, 9 months, 24 days, at 6 %, amount to \$161.35?
 This problem comes under Art. 322.
 The principal thus found is \$145.50;
 Hence, the discount is \$161.35—145.50, or
 \$15.85+

2. A man owed \$1200 due in 6 months and 20 days. What sum would pay the debt immediately, money being worth 6 %?

3. Find the discount on a debt due in 4 months and 10 days, the debt being \$450, and money being worth 7 %?

4. How much would a man save by discharging a debt of \$1125 due in 1 year, 2 months, 18 days, interest at 5 %?

5. What is the difference between the bank and the true discount on \$100 due in 1 year, at 6 % after maturity? On \$1000? On \$10000?

378. To find what is still due after discounting part of a debt.

1. If you owed \$1000 due in 1 year, and should be allowed $12\frac{1}{2}$ % for ready money, what would you still owe after paying \$700 immediately?

Suggestion.— $12\frac{1}{2}$ % is what part of 100 %?
 What part of the debt does the money paid discharge?
 If \$700 is $\frac{1}{4}$ of what it pays off, what remains unpaid?

2. A man owed \$1600 ; but being offered $2\frac{1}{2}\%$ discount for cash, immediately raised and paid \$780. How much remained unpaid? Ans. \$800.

Suggestion. — Prove this answer.

PARTIAL PAYMENTS.

379. Partial Payments consist of money paid at different times in discharge of part of a debt.

380. These payments are acknowledged by receipt, or by endorsement on the back of the note or other obligation.

381. In this class of problems two elements must always be carefully considered :

- a. The amount of each payment.
- b. The amount of interest accrued at the time of each payment.

382. There may arise two essential conditions :

- a. Any payment may exceed the interest due at the time that payment is made.
- b. The interest due at the time a payment is made may exceed that payment.

383. These conclusions follow, and must be carefully observed in applying the rule :

- a. If the interest due at the time a payment is made is *less* than that payment, the new principal will be *less* than the debt.
- b. If the interest due at the time a payment is made is *greater* than that payment, the new principal will be *greater* than the debt.

- c. To avoid compound interest the new principal must *never* be greater than the debt. Hence, the following rule :

UNITED STATES RULE.

I. *Compute the interest from the date of the note to the time of the first payment.*

II. *If this interest is less than this payment, add the interest to the principal and subtract the payment from this amount. The remainder will form a new principal.*

III. *But, if the interest is greater than the payment, find the interest on the principal up to a time when the sum of two or more payments exceeds the interest due at that time.*

IV. *Add the interest to the original, or to the new principal, as the case may require, and subtract the sum of the payments. The remainder will form another new principal. Proceed with remaining payments as before.*

EXAMPLES.

1.

ST. LOUIS, Mo.,

\$1200.

January 1, 1880.

One year after date I promise to pay to the order of Herr Maim, twelve hundred dollars, with interest from date at the rate of eight per cent. per annum. Value received, and payable without defalcation or discount.

T. T. TAINTER.

On this note the following payments were made :

April 1, 1880, \$150 ; June 1, 1880, \$50 ; September

1, 1880, \$20; October 1, 1880, \$500. What was due at maturity?

Suggestion. — Date of maturity January 4, 1880.

Find the difference of time between January 1, and April 1, 1880.

Find the interest on \$1200 for this period.

Does this interest exceed the payment, \$150?

Then, what is the new principal?

Find the difference of time between April 1, and June 1, 1880.

Find the interest on \$1074 for this period.

Does this interest exceed the payment, \$50?

What is the new principal?

Find the difference of time between June 1, and September 1, 1880.

Find the interest on \$1038.32 for this period.

Since the interest is \$20.7664, we must compute the interest on \$1038.32 from June 1 to October 1, 1880, or 4 months.

What is the interest for this period? What the amount? What the sum of the last two payments? What the new principal?

What time has the note still to run? From October 1, 1880, to January 4, 1881.

What is the interest? What the amount?

2. On a note for \$1000, dated May 15, 1879, and due in 9 months, are found the following endorsements: June 1, \$25; August 5, \$40; September 10, \$10; October 20, \$400. What was due at maturity?

3. On a note for \$2000, dated December 10, 1878, and payable in 12 months, are found the following endorsements:

January 27, 1880, \$49; February 5, \$104; May 16,

\$60; July 21, \$700. This note was not paid until 6 months after maturity. What was due then?

NOTE. — When no rate is mentioned, 6 % is generally understood.

4. A note for \$3800 was given May 8, 1871. On December 1, 1871, \$120 was paid; March 6, 1872, \$500; November 19, 1872, \$75. What was due December 30, 1872?

5. A note for \$1800 was given March 1, 1865. Payments were made May 1, 1866, \$200; June 16, 1867, \$300; March 1, 1869, \$280. What was due October 16, 1870?

384. There are different rules for partial payments. They give results differing slightly from each other.

On a debt of \$1000 due in 1 year, at 6 %, and on which a payment of \$500 is made at the end of 6 months, the results are, by these several rules :

By the United States Rule, ... \$545.90

By the Present Worth Rule, ... \$545.44

By the Vermont Rule, ... \$545.00

The object of the rule for partial payments is to avoid giving either debtor or creditor any advantage.

NOTES. — BY THE PRESENT WORTH RULE. — The \$500 paid at the end of 6 months was *virtually* the same as paying \$485.44 on the day the debt was received. Hence, the amount to be paid at the end of the time is \$1000 minus \$485.44, which is \$514.56, increased by 6 % of \$514.56 for 1 year. That is, the actual amount of credit enjoyed by the debtor for 1 year, was the virtual use of \$514.56 for that time. Hence, he owed at the expiration of that time the amount of \$514.56, at 6 %, which is \$545.44. This method avoids compound interest in favor of either party. The calculations are made on the basis of *simple interest*.

By the VERMONT RULE, the amount of the debt, \$1000, for 1 year, *minus* the amount of the virtual payment, \$485.44, for 1 year, gives \$545.44 as the sum due at the end of the year. And by compounding the virtual payment, \$485.44, at the end of 6 months, we shall have at the end of the year the sum found due by the Vermont Rule.

By the UNITED STATES RULE, if we compound \$1000, and also \$485.44, in 6 months, at the end of the year we shall have the result given by the United States rule.

By the Vermont Rule, then, compound interest is allowed on the **virtual** payment, and thus favors the debtor. And by the United States Rule interest is compounded on both principal and virtual payment

EQUATION OF PAYMENTS.

385. Equation of Payments is the process of finding the time at which different sums of money, due at different times, may be paid at once without loss to debtor or creditor.

386. The Term of Credit is the time which must elapse before a debt becomes due and payable.

387. The Average Term of Credit is the number of days or months, at the end of which the several debts due at different times, may all be paid at once without loss to either party.

388. The Equated Time is the date at which the several amounts due at different times, may be paid in a single payment.

a. Mr. A may owe Mr. B several debts, due at different times.

b. Mr. B may owe Mr. A several debts, due at different times.

c. Mr. A may owe Mr. B, and, at the same time,

Mr. B may owe Mr. A, several debts, due at different times.

389. To find the equated time of payment when several debts are due at different times, from one man to another.

1. Mr. A owes Mr. B as follows: \$200 due in 3 months; \$300 due in 5 months; \$250 due in 2 months; \$150 due in 6 months. When may these several debts be paid at once?

Solution.

\$200 due in 3 months is the same as \$1 due in 600 months.

\$300 due in 5 months is the same as \$1 due in 1500 months.

\$250 due in 2 months is the same as \$1 due in 500 months.

\$150 due in 6 months is the same as \$1 due in 900 months.

\$900 due in $3\frac{1}{3}$ months is the same as \$1 due in 3500 months.

Hence, the entire debt, \$900, may be paid in $3\frac{1}{3}$ months, without loss to either party.

390. Under *b*. there are four possible suppositions:

1. { The balance of account may fall on the debit side;
and,
The balance of the term of credit may fall on the debit side.
2. { The balance of account may fall on the credit side; and,
The balance of the term of credit may fall on the credit side.
3. { The balance of account may fall on the credit side; and,
The balance of the term of credit may fall on the debit side.

4. { The balance of account may fall on the debit side ;
and,
The balance of the term of credit may fall on the
credit side.

391. The first and second suppositions show the same fact, viz. : *That the equated time is earlier than the assumed date.*

392. The third and fourth suppositions show the same fact, viz. : *That the equated time is later than the assumed date.*

393. The **Assumed, or Focal Date,** is a date taken at pleasure, from which to compute the several terms of credit. It is generally more convenient to assume either the first or the last date in the account, as the focal date.

394. To find the equated time when both debit and credit items are involved.

1. On January 1, 1880, Mr. James bought goods of Mr. Jones as follows :

A bill for \$100 due in 3 months ;

A bill for \$300 due in 4 months ;

A bill for \$500 due in 6 months.

On February 1, 1880, Mr. Jones bought of Mr. James the following :

A bill for \$150 due in 1 month ;

A bill for \$250 due in 2 months ;

A bill for \$350 due in 3 months.

Find the equated time for settlement.

Solution.

1. $\left\{ \begin{array}{l} \$100 \text{ due in 3 months is the same as } \$1 \text{ due in } 300 \text{ months.} \\ \$300 \text{ due in 4 months is the same as } \$1 \text{ due in } 1200 \text{ months.} \\ \$500 \text{ due in 6 months is the same as } \$1 \text{ due in } 3000 \text{ months.} \\ \hline \$900 \text{ due Mr. James. Jones' term of credit is } 4500 \text{ mo. on } \$1. \end{array} \right.$
2. $\left\{ \begin{array}{l} \$150 \text{ due in 1 month is the same as } \$1 \text{ due in } 150 \text{ months.} \\ \$250 \text{ due in 2 months is the same as } \$1 \text{ due in } 500 \text{ months.} \\ \$350 \text{ due in 3 months is the same as } \$1 \text{ due in } 1050 \text{ months.} \\ \hline \$750 \text{ due Mr. Jones. James' term of credit is } 1700 \text{ mo. on } \$1. \end{array} \right.$
3. Balance in favor of James, \$150. Term of credit in favor of Jones, 2800 months.
4. Since Mr. Jones owes Mr. James a balance of \$150, but is entitled to the use of \$1 for 2800 months, he is entitled to the use of the balance due Mr. James, $2800 \div 150$, or $18\frac{2}{3}$ months. Hence, the time of settlement is July 21, 1881.

395. In an account kept with a customer, he is charged, or debited, with the amount of each purchase, and credited with the amount of each payment made. Such an account must show one of the three conditions *a*, *b*, *c*. ART. 390.

396. The conditions shown under *a*. and *b*. have been considered under ART. 389.

397. The conditions shown under *c*. ART. 388, show two of the four facts in ART. 390. And the solution of any problem of this class consists in interpreting these two facts.

398. When the account is on A's books, and against B.

1. A balance of account on the debit side shows that B owes A.

2. A balance of account on the credit side shows that A owes B.

3. A balance of term of credit on the debit side shows A owes B interest.

4. A balance of term of credit on the credit side shows B owes A interest.

399. When A owes B a balance of account, and interest too, the balance of account should have been paid earlier. Also, when B owes A a balance of account, and interest too, the balance of account should have been paid earlier.

400. When A owes B a balance of account, but B owes A interest, the balance of account should be paid later. Also, when B owes A a balance of account, but A owes B interest, the balance of account should be paid later. Hence, the following

RULE FOR FINDING EQUATED TIME :

I. Assume the earliest or latest date as the true date for settlement, and compute the terms of credit in favor of each party.

II. Divide the balance of the term of credit by the balance of indebtedness, the quotient will indicate the time before or after the assumed date, at which settlement should be made.

III. When both balances fall on the same side of the account, count forward; but when the balances fall on opposite sides, count backward, from the assumed date.

1. Find the equated time for settling the following account on Mr. A's books. Mr. B's account stands :

DR.			Mr. B.			CR.		
1880.			1880.					
April 5	To mdse. on 2 mo.	\$300 00	April 20	By cash....	\$410 00			
May 15	“ cash.....	200 00	June 5	“ mdse..	120 00			
June 10	“ mdse.....	800 00	July 15	“ mdse..	360 00			

B owes A.....\$1300.

A owes B.....\$890.

Balance favor of A.....\$410.

Balance of credit favor of A, 12140 days on \$1.

Balance of credit favor of A, 30 days on \$410.

Finally, the balance of account shows that B owes A.

The balance of term of credit shows B owes A interest.

Hence, the balance of account, \$410, should have been paid 30 days earlier than the assumed date, which, in this case, was July 10.

The account could have been settled June 15, 1880.

2. On Mr. B's books Mr. A's account stands :

DR.			Mr. A.			CR.		
1880.			1880.					
June 1	To mdse. on 3 mo.	\$450 20	July 3	By cash...	\$400 00			
July 7	“ mdse.....	375 50	Sept. 5	“ cash...	50 50			
Aug. 20	“ mdse.....	180 00	Oct. 10	“ cash...	75 00			
Sept. 25	“ mdse.....	250 75	Nov. 15	“ cash...	150 00			

What is the time of settlement? What the balance due? What must be the face of a note for 60 days, given in settlement?

3. Mr. Joseph Mason holds two notes against Mr. John Craig. The first note is for \$1500, dated January 5, 1880, and due in 12 months. The second note is for \$2100, dated January 5, 1880, and due in 18 months. Mr. Craig wishes to give a new note, dated July 1, and take up the other two. What time should it run?

Ans. 9 mo., 19 days.

4. A owes B \$500, due in 10 mo. B owes A \$1000. due in 5 mo. A owes B \$300, due in 8 mo. B owes A \$600, due in 4 mo. When may they settle?

TAXES.

401. A tax is an assessment on property or person for government purposes.

402. A tax on property is a **property-tax**. A tax on individuals is a **poll-tax**.

403. A property-tax is levied on the estimated value of the property, at a certain rate per cent.

404. A **poll-tax** is a certain sum assessed against each individual liable to taxation.

405. In levying a tax on property the following elements are considered :

- a. The value of the property taxed.
- b. The amount of taxes to be raised.
- c. The rate of taxation.

406. Given the valuation and the amount to be raised on the property to find the rate of taxation.

1. On property valued at \$345639, a tax of \$1728.19 $\frac{1}{2}$ is to be raised. Find the rate. Ans. .005.

Suggestion. — This problem is similar to those under ART. 349.

2. What is the rate of taxation when \$435.39 is levied against property valued at \$72565?

407. Given the valuation and the rate of taxation to find the tax.

1. On property valued at \$1276841, a tax is assessed at the rate of 4 mills on a dollar. What is the amount of the tax?

Suggestion. — This problem is similar to those under ART. 348.

2. When the valuation is fixed at \$456892, what amount of tax can be raised at $4\frac{1}{2}$ mills on each dollar?

408. Given the tax to be raised and the rate of taxation to find the valuation.

1. What must be the valuation that a tax levied at the rate of 3 cents on a \$100 may reach \$3806.868?

Suggestion. — This problem is similar to those under ART. 357.

2. In a village, the tax to be raised for public purposes is \$2875.20, and the rate is .005. Find the valuation of the property.

3. In the last example, what is Mr. A's tax, who is assessed \$976? Mr. B's, who is assessed \$1106? And Mr. C's, who is assessed \$1319?

4. Make a tax-table showing the tax from \$1 to \$10, on \$100, \$500, \$1000, when property valued at \$27000 is assessed \$67.50?

DUTIES.

409. Duties are indirect taxes levied upon goods imported or exported.

410. Duties are either

a. *Ad valorem*, and estimated on the value of the goods, or

b. Specific, and estimated on the weight of the goods.

411. Certain allowances, known as **breakage, leakage, tare, tret**, are deducted before duties are levied.

412. Before these allowances are made, the weight

of the goods, or the value of the goods, is known as **gross weight**, or **gross value**.

413. After these allowances are made, the weight of the goods, or the value of the goods, is known as **net weight**, or **net value**.

414. The terms breakage, leakage, sufficiently indicate the nature of these allowances.

415. **Tare** is an allowance made for the weight of the bags or boxes which contain the goods. **Tret**, an allowance made in weight on account of waste, or refuse matter.

416. To compute ad valorem duties.

Suggestion. — Compute the duty on the *net value*, at the given rate.

1. A merchant invoiced 70 pieces of fine silk, each containing 60 yards, at \$2.75 a yard. What was the duty, at $37\frac{1}{2}\%$?

2. Find the duty on 27 cases of goods, each containing 532 yards, invoiced at \$1.37 $\frac{1}{2}$, the duty being $24\frac{3}{4}\%$?

417. To compute specific duties.

Suggestion. — Compute the duty on the *net weight*, at the given rate.

1. When the duty on tea is 9 cents per pound, what duty must be paid on 50 chests of Y. H., each weighing 125 lbs., tare $12\frac{1}{2}\%$?

2. Dodd, Brown & Co. imported 30 cases of goods, each weighing 658 lbs. An allowance of 8% being allowed for tare, and the duty being 40%, what did they pay in gold, when gold was at \$1.50?

3. When gold was quoted at 280, a merchant imported 21 cases of goods, each weighing in gross 710 lbs. Tare being 10 %, and the duty $37\frac{1}{2}$ %. What duty did he pay?

MONEY, STOCKS AND BONDS.

418. Money is a general medium for effecting exchanges.

419. Stocks are shares in any enterprise carried on under a corporate name.

420. A bond is a written, or printed obligation, securing the payment of a specified sum of money, at a specified time.

421. A great variety of stocks and bonds are now dealt in :

- a. Bank stocks, which constitute bank capital.
- b. Mining stocks, of various kinds.
- c. Railway stocks, and railway bonds.
- d. Government bonds.
- e. State, city, county, township bonds.

422. These are bought and sold either at par; at a discount, or at a premium.

423. Stocks, or bonds are at par when they bring their face value ; at a discount when they bring less than their face value ; and at a premium when they bring more than their face value.

424. Commissions are estimated on the par value of stocks or bonds.

425. Market value is the present value of bonds or stocks.

426. Stocks and bonds are quoted at a certain rate per cent., which indicates whether they are above, at, or below par.

427. In problems of this class four elements are considered :

- a.* The par value of the stock or bond.
- b.* The market value of the stock or bond.
- c.* The rate of premium or discount.
- d.* The amount of premium or discount.

428. Hence, there are six cases.

429. Given the par value and the rate to find the premium or discount.

1. What is the market value of currency when it is at a discount of $37\frac{1}{2}\%$?

Suggestion. — This problem is similar to those under ART. 348.

2. When gold is at a premium of 25 %, how much currency will buy \$1250 in gold?

3. When U. S. 6's are quoted at 125, what must I pay for \$3560 in bonds?

4. When N. Y. Central is quoted at 121, what must I pay for 20 shares, the par value being \$100?

5. When Erie is at $37\frac{1}{2}$, what will 2500 shares cost? Par value \$100.

430. Given the par value and the amount of premium or discount to find the rate per cent.

1. For 20 shares of railway stock a man paid \$1750. What was the rate of discount?

Suggestion. — This problem is similar to those under ART. 349.

2. If 50 shares of bank stock were bought for \$4000 cash, what was the stock quoted at?

Ans. 80 %.

3. 20 shares of bank stock, whose par value was 100, was offered for \$1250. What was it quoted at on the market?

Ans. $37\frac{1}{2}$ %.

431. Given the premium, or discount, and the rate to find the par value.

Suggestion. — This problem is similar to those under ART. 350.

1. At 5 % discount, 40 shares of railway stock cost \$3800. What was the par value of the stock?

Ans. \$100.

2. Sold 40 shares of \$100 each, at a discount of \$40. Find the market value, and the rate of discount.

Rate 2 %.

3. A man sold 300 shares of bank stock, at a discount of \$4500. What was the rate, and what its market value?

Rate 2 %.

4. An operator made a clear profit of \$2600 on 100 shares of railway stock. What was it quoted?

Ans. 126 %.

432. Given the market value and the rate to find the par value.

1. Seventy-five shares of bank stock sold for \$7000, at a discount of $6\frac{2}{3}$ %. What was the par value of each share?

Suggestion. — This problem is similar to those under ART. 350.

2. If the market value of stock at 5 % premium is \$2520, what is its par value?

Ans. \$2400.

3. Desiring money, a man sold a note he held against another, at 12 % discount, for \$2563.44. What was the face of the note? Ans. \$2913.

4. A promissory note was sold at 15 % discount, for \$11223.40. Find the face of the note, and the discount.

433. To find the premium on gold when its value in currency is known.

1. When a merchant is obliged to pay \$2750 for \$2000 in gold, what is the rate of premium on gold?

Suggestion. — What is the amount of premium paid?

\$750 is what per cent. of \$2000? See ART. 349.

2. If a man paid \$150950 in currency for \$150200 in gold, what was the rate of premium? Ans. $\frac{1}{2}$ %.

434. Given the premium and rate to find the amount of the investment in gold, or in currency.

1. When gold is quoted at 108, the premium on a gold investment amounts to \$97.28. What amount of gold was purchased?

Suggestion. — What is the premium on \$1 in gold?

Then, \$97.28 is the premium on how many dollars in gold?

2. A merchant paid a premium on gold, amounting to \$1677.87, when gold was quoted at 109. What amount did he buy? Ans. \$18643.

435. Given the investment, in currency, and the rate to find the investment in gold.

1. What amount of gold can be purchased for \$12339.60 in currency, when gold is quoted at 104?

Suggestion. — What is the cost of \$1 in gold?

Then, \$12339.60 will buy how many dollars in gold?

2. When the premium on gold is $5\frac{1}{2}\%$, what amount of gold can be bought for \$12660 in currency?

3. The rate on gold being $12\frac{1}{2}\%$, how much gold can be bought for \$27189? Ans. \$24168?

436. Given the rate of premium on gold to find the rate of discount on currency.

1. When the rate of premium on gold is 50 %, what is the rate of discount on currency?

Suggestion. — \$1 in gold costs how much currency?

What is the loss on the money invested?

Then, what is the rate of loss on the investment? Ans. $33\frac{1}{3}\%$.

2. When gold is quoted at $133\frac{1}{3}$, what is the rate of discount on currency? Ans. 25 %.

437. To find the currency value of goods bought on a gold basis.

1. A merchant paid 40 cents a yard for goods in London, when gold was quoted at $112\frac{1}{2}$. Find their value in currency.

Suggestion. — The premium is what *part* of the cost in currency?

The cost in gold is what *part* of the cost in currency?

Then, what is the cost in currency?

Ans. 45 cents.

2. When silk is valued at \$5.75 a yard on a gold basis, what must it be valued a yard on a currency basis, gold being at 115?

3. Goods that cost 75 cents a yard on a gold basis, are worth what on a currency basis, when gold is at $106\frac{2}{3}$?

4. A merchant bought goods in Liverpool at $6\frac{1}{2}$ cents a yard, and paid duties in this country at the rate of 20 %, in gold. After paying 2 % for freight, what must he mark the goods to realize a net profit of $33\frac{1}{3}$ %?

Ans. $91\frac{8}{25}$ cents.

MISCELLANEOUS PROBLEMS.

1. Sold bank stock at 95, and lost \$950. Find the market value, and the par value.

2. When 240 shares of bank stock brought \$10800, discount being at the rate of 5 %, how much was lost if the stock was bought at a premium of 15 %?

3. A broker made a purchase for a customer when railway stock was at 70, and charged $1\frac{1}{2}$ % brokerage. He afterwards sold when the stock stood at 80, brokerage $1\frac{1}{3}$ %. My gain being \$754.37 $\frac{1}{2}$, how many shares did I buy, if the par value was \$25?

4. What must a 6 % bond cost to yield 9 % on the investment?

5. What must a 5 % bond cost to yield 8 % on the investment?

6. What must a $4\frac{1}{2}$ % bond cost to yield 4 % on the investment?

7. What must a 5 % bond cost to yield 4 % on the investment?

8. Which is the more profitable investment, a 5 % bond at 95, or a 6 % bond at 90? What per cent.?

9. Sold U. S. 6's at 104, and invested in U. S. 7-30's at 90. What per cent. did I gain or lose?

PARTNERSHIP.

438. Problems in partnership involve three elements.

- a. The number of partners, or persons interested.
- b. The amount of capital each partner invests.
- c. The time the capital was employed.

439. Simple Partnership involves only the first two elements, the number of partners, and the amount of capital of each.

1. A and B engaged in speculation and gained \$2100. A's share of the capital was \$4000, and B's share \$3000. Find the gain of each

Suggestion. — What was the entire capital invested?

A's was what part of the entire capital?

B's was what part of the entire capital?

Then, A's gain was what *part* of the entire gain?

And B's gain was what *part* of the entire gain?

440. From this analysis we derive the following rule for simple partnership:

RULE. — I. *Find the entire capital employed.*

II. *Find what part each man's capital is of the entire capital.*

III. *Take such part of the entire gain or loss for each partner, as his capital is of the entire capital.*

2. A, B, C, and D gain \$2700 in trade. A's money was \$200, B's was \$275, C's, \$325, D's, \$100. Find the gain of each.

3. Two men engaged in trade for a certain time found they had lost \$750. The investment of the first was twice as great as that of the second, whose capital was \$2500. How much capital did each have at the close of the business?

First, \$4500; second, \$2250.

4. Divide 2678 in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Suggestion. — What whole numbers are in the ratio of the given fractions?

5. Divide \$4400 among A, B, C, D, E, and F, in the ratio of the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{1}{12}$.

NOTE. — For further exercises see analysis, ART. 579.

441. Compound partnership involves the three elements: Number of partners, the amount of capital invested by each, and the time each one's capital was invested.

1. Two men gained in business \$2300. The capital of the first, \$700, was employed 3 months; the capital of the second, \$500, was employed 5 months. What was each man's share of the gain?

Suggestion. — \$700 for 3 mo. equals what sum for 1 mo?

\$500 for 5 mo. equals what sum for 1 mo?

What was the entire capital for 1 mo.?

What part of the entire investment for 1 month was each man's investment for 1 month?

Then, what was each man's gain?

442. From this analysis we derive the following rule for compound partnership:

RULE. I. — *Find each man's investment for one month.*

II. *Find the entire investment for one month.*

III. *Take such part of the gain or loss, as each man's investment for one month is of the entire investment for one month.*

443. Any investment bears the same relation to any other investment that the profit, or loss, on the first investment bears to the profit, or loss, on the second investment, when the investments are made for the same length of time, at the same rate of profit. This fundamental principle underlies all problems in partnership.

2. If four men, engaged in trade, gain \$1480, what is the remaining capital of each at the end of the transaction, if the first man invested \$900 for 2 months, the second man \$700 for 3 months, the third man \$1100 for 4 months, and the fourth man \$1300 for 5 months?

3. A, whose investment was for 6 months, received \$420 profit; B, whose investment was for 7 months, received \$280 profit. If the whole investment was \$3300, what sum did each man invest?

EXCHANGE.

444. Exchange is the process of paying a debt due in the money of one country, in an equivalent value expressed in the money of the other country.

445. Exchange is either **Domestic**, or **Foreign**.

446. Payments made at distances are effected by means of a *draft*. When drawn on a foreign country, the draft is known as a **Bill of Exchange**.

447. A draft, or bill requiring payment when pre-

sented, is a **Sight Draft**. One requiring payment in a specified time, is a **Time Draft**, or **Bill**.

448. The original parties to a bill are the **Maker**, the **Drawee**, the **Payee**. Others may become parties by indorsement, or by acceptance after protest.

449. **Acceptance** is necessary only in the case of time drafts.

450. An **Acceptance** consists in the drawee, or party on whom the bill is drawn, writing his name across the face of the bill, together with the date of acceptance. This is generally done in red ink.

451. An acceptance fixes the legal obligation of the drawee, who, before acceptance, is not bound to pay the bill at maturity.

452. The **Par of Exchange** is based on the relative value of the money of different nations.

453. The relative value of the money of different nations depends

a. On the **intrinsic** value of their coins, determined by their weight and purity.

b. On the **commercial** value of their coins; that is, on their market value.

454. **Rate of Exchange** is the current price of a draft at the place where it is purchased.

455. The rate of exchange depends on a variety of conditions, and is almost constantly varying.

456. **Direct Exchange** is effected between only *two* countries; **Indirect Exchange** between *three* or more.

457. Indirect Exchange is called **Arbitration of Exchange**; and consists in making remittances from one country to another, by the way of other countries.

458. Given the rate to find the cost of a sight draft.

1. What is the cost of a sight draft on N. Y. for \$1000, the rate of exchange being $1\frac{1}{2}\%$ above par?

Suggestion. — What is the cost of \$1, at $1\frac{1}{2}\%$ premium?

What is the cost of \$1000, at the same rate?

2. Find the cost of a sight draft on San Francisco for \$2500, when the rate is $\frac{3}{4}\%$ discount.

459. Given the time and rate to find the cost of a time draft.

1. What is the cost of a draft on N. Y. for \$2000, bought at St. Louis, and payable 60 days after date, when money is worth 8 %, and exchange is $\frac{7}{8}\%$ discount?

Suggestion. — What is the present worth of \$1 due in 63 days, at 8 % discount?

Then, what will \$1 of the draft cost, at $\frac{7}{8}\%$ discount?

And, what will \$2000 of the draft cost?

2. A draft on Boston for \$1500, payable in 30 days, is bought when the rate of exchange is $\frac{1}{2}\%$ premium. Find its cost.

460. Given the rate, time, and cost of a time draft to find its face.

1. A merchant paid \$3200 for a draft on N. Y., payable in 60 days, when the discount was $1\frac{7}{8}\%$. Find the face of the draft.

Suggestion. — What is the present worth of \$1, due in 63 days, at $1\frac{7}{8}\%$ discount?

Then, \$3200 is the present worth of how many dollars.

2. Sent an agent in New Orleans a draft for £15000, payable in 30 days, premium at $\frac{3}{8}\%$, to be invested in cotton at $11\frac{1}{2}$ cents, his commission being $2\frac{1}{2}\%$. How much cotton did I purchase? What did the draft cost?

3. A merchant in N. Y. wishing to remit £3840 to London, finds that a pound sterling is quoted at \$4.86, and gold at 110. What must he pay for a draft to cover the amount?

4. What must be the face of a sight draft for £1500, when sterling exchange is \$4.84 $\frac{1}{2}$, and gold at 101 $\frac{3}{4}$?

What would be the face of this draft, if drawn for 90 days?

ALLIGATION.

461. Alligation treats of compounding ingredients of different values.

a. So as to obtain the average value of the mixture; or

b. So as to determine the ratio in which the quantities may be mixed to yield a given average.

462. Alligation Medial is the process of determining the average price when the different prices of the several ingredients are known.

463. Alligation Alternate is the process of determining

a. The ratio in which the ingredients must be mixed to yield a given ratio.

b. The quantity of the other ingredients taken when one of the ingredients is limited.

- c. The quantity of each ingredient when the mixture is limited.

464. Given the quantity of each ingredient and the price of each to find the average price.

1. A grocer mixed sugars at different prices, as follows: 25 lbs. at 6 cents, 40 lbs. at $6\frac{1}{2}$ cents, 50 lbs. at 8 cents, 15 lbs. at 4 cents, 20 lbs. at 9 cents. At what price must he sell the mixture?

Suggestion. — Find the cost of 25 lbs. at 6 cents.
 Find the cost of 40 lbs. at $6\frac{1}{2}$ cents.
 Find the cost of 50 lbs. at 8 cents.
 Find the cost of 15 lbs. at 4 cents.
 Find the cost of 20 lbs. at 9 cents.
 Then the cost of 150 lbs. was 1050 cents.
 What must be the price of *one* pound?

2. Find the average price of the following mixture: 10 lbs. of tea, at 70 cents, 20 lbs. at 80 cents, 35 lbs. at \$1.10?

3. At what price per pound can a mixture of 20 lbs. of butter at 40 cents, 10 lbs. at 30 cents, 5 lbs. at 15 cents, and 2 lbs. at 10 cents, be sold so as to neither gain nor lose by the operation?

4. Find the average price of 100 lbs. of flour at $3\frac{1}{2}$ cents, 30 lbs. of meal at $\frac{1}{3}$ of a cent, and 15 lbs. of bran at $\frac{1}{4}$ of a cent? Ans. $2\frac{1}{2}$ cts. *nearly*.

465. Given the prices of the several quantitles, and the average price, to find the ratio of the mixture.

1. In what ratio must sugars at 8 cents, 10 cents, 12 cents, and 15 cents a pound, be mixed to make the mixture worth 11 cents a pound?

OPERATION.

Av. price, 11 cents.	On 1 lb. worth 8 cts., but sold at 11 cts., there is a <i>gain</i> of 3 cts. Hence, to gain only 1 cent $\frac{1}{3}$ of a pound must be taken.	Gain, 1 cent.
	On 1 lb. worth 10 cts., but sold at 11 cts., there is a <i>gain</i> of 1 ct. Hence, to gain only 1 cent 1 pound must be taken.	Gain, 1 cent.
	On 1 lb. worth 12 cts., but sold at 11 cts., there is a <i>loss</i> of 1 ct. Hence, to lose only 1 cent 1 pound must be taken.	Loss, 1 cent.
	On 1 lb. worth 15 cts., but sold at 11 cts., there is a <i>loss</i> of 4 cts. Hence, to lose only 1 cent $\frac{1}{4}$ of a lb. must be taken.	Loss, 1 cent.

Thus, by taking $\frac{1}{3}$ of a pound at 8 cents, and 1 pound at 10 cents, there is a gain of 2 cents.

But, by taking 1 pound at 12 cents, and $\frac{1}{4}$ of a pound at 15 cents, there is a loss of 2 cents.

Now, any integers in the ratio of $\frac{1}{3}$, 1, 1 and $\frac{1}{4}$, will represent the *relative* number of pounds of each ingredient to be taken to make a mixture worth 11 cents a pound.

We may take 4 lbs. at 8 cents, 1 lb. at 10 cents, 1 lb. at 12 cents, and 3 pounds at 15 cents.

Proof.—4 lbs. at 8 cents are worth 32 cents.

1 lb. at 10 cents is worth 10 cents.

1 lb. at 12 cents is worth 12 cents.

3 lbs. at 15 cents are worth 45 cents.

Then, 9 pounds are worth..... 99 cents.

And 1 pound is worth 11 cents, the average price.

NOTE—This class of problems admits of a variety of solutions, each solution giving a different result, but all the results satisfying the conditions of the problem.

466. The principle involved in ART. 265 underlies ARTS. 467 and 468.

467. To find the quantity of each ingredient when the quantity of one is limited.

1. A grocer mixed 72 pounds of sugar at 12 cents, with sugars at 4, 5 and 9 cents a pound, so as to sell the mixture at 8 cents. How many pounds of each did he take?

Suggestion. — By ART. 465 we find the ratio of the mixture to be as follows: 3 lbs. at 4 cts., 4 lbs. at 5 cts., 12 lbs. at 9 cents., and 3 lbs. at 12 cents.

But, as there are 24 times 3 lbs. at 12 cts., there must be in the mixture 24×3 , or 72 lbs. at 4 cts.; 24×4 , or 96 lbs. at 5 cts.; 24×12 , or 288 lbs. at 9 cts.

NOTE. — Let the pupils find *five* other answers, and prove them.

468. To find the quantity of each ingredient when the quantity of the mixture is limited.

1. What are the respective quantities of sugar at 4, 8, 10 and 12 cents a pound, in a mixture of 200 pounds, worth 9 cents a pound?

Suggestion. — Find, by ART. 465, the integral ratios.

Then, by ART. 440, divide 200 in the ratio of these numbers.

469. A large class of problems may be solved by a judicious application of the principles involved in ARTS. 465, 466, 467, 468. They are, however, of little practical importance.

MISCELLANEOUS PROBLEMS.

2. A man bought 200 bushels of grain at 90 cents, 150 at 70 cents, and 250 at \$1. He wishes to pur-

chase a sufficient number of bushels at 50 cents, to enable him to sell the lot at an average price of 75 cents a bushel, and realize a profit of 25 %. How many bushels must he buy? Ans. 1750.

3. A speculator bought land at an average price per acre, of \$75. His investment amounted to \$18750. If the land cost him \$60, \$70, \$80 and \$85 an acre, how many acres at each of these prices did he buy?

4. At what price per bushel must a mixture of wheat consisting of 45 bushels worth 95 cts., 30 bushels worth \$1, and 25 bushels worth \$1.05, be sold to realize a profit of 5 per cent?

POWERS AND ROOTS.

470. Finding any power of a number is called **involution**.

471. The first power of a number is the number itself.

472. The **Second Power** of a number is the **Product** found by taking the number **twice** as a factor.

473. The **Third Power** of a number is the **product** found by taking the number **three** times as a factor

474. Any **Power** of a number is the **product** found by taking that number as many times as a factor as there are units in the index of the power.

475. The index of a power is a figure placed at the right and a little above the number. Thus :

2^2 means the second power of 2, or 2×2 .

3^3 means the third power of 3, or $3 \times 3 \times 3$.

4^4 means the fourth power of 4, or $4 \times 4 \times 4 \times 4$.

Numbers.	1,	2,	3,	4,	5,	6,	7,	8,	9.
Squares.	1,	4,	9,	16,	25,	36,	49,	64,	81.

Hence, the square, or second power of any number consisting of *one* figure, cannot consist of more than *two* figures.

Numbers.	10,	20,	30,	40,	50,	60,	70,	80,	99.
Squares.	100,	400,	900,	1600,	2500,	3600,	4900,	6400,	9801.

Hence, the square of a number consisting of *two* figures cannot consist of more than *four* figures.

476. To involve, or raise a number to any power consists in taking the number a given number of times as a factor.

477. The second power is called the square.

478. The third power is called the cube.

EXERCISES.

1. Find the first five powers of 2; of 3; of 4.
2. Find the first five powers of 5; of 6; of 7.
3. Find the third power of 8; the fourth power of 9.
4. Find the fifth power of 10; the sixth power of 11.

Find the powers indicated below:

5. $(12)^3$, $(13)^4$, $(14)^3$, $(15)^4$, $(16)^3$, $(17)^3$, $(18)^2$, $(19)^2$, $(20)^2$.

6. Find the square of 21, 23, 25, 27, 29.

7. Find the cube of 22, 24, 26, 28, 30.

8. Square 31, 33, 35, 37, 39, 41, 43, 45.
9. Cube 32, 34, 36, 38, 40, 42, 44, 46.
10. Square 1, 10, 100, 1000, 10000, 100000.
11. Cube 1, 10, 100, 1000, 10000, 100000.
12. What is the square of 101, 103, 105, 107, 109, 111?
13. What is the square of 102, 104, 106, 108, 110, 112?
14. What is the cube of 22, 44, 55, 66, 77, 88, 99?
15. What is the cube of 62, 72, 82, 92?

479. To find any power of a fraction.

1. Find the square of $2\frac{1}{2}$; the cube of $2\frac{1}{4}$; the fourth power of $2\frac{3}{4}$.

Suggestion. — This is the case of multiplying a fraction by a fraction.

2. Square $3\frac{1}{2}$, $3\frac{1}{8}$, $3\frac{2}{8}$, $4\frac{1}{4}$, $4\frac{1}{8}$, $4\frac{1}{2}$, $4\frac{3}{8}$, $4\frac{3}{4}$.
3. Cube $4\frac{1}{4}$, $4\frac{1}{8}$, $4\frac{1}{2}$, $4\frac{2}{8}$, $4\frac{3}{8}$, $4\frac{1}{8}$, $4\frac{5}{8}$.
4. Square .5, .7, .08, .09, .011, .0012.

Suggestion. — This is the case of multiplying a decimal by a decimal.

5. Cube .2, .02, .002, .22, .022, .222.
6. Cube .25, .125, .1235, 1.235, 12.35, 123.5.
7. Square 12345.6, 1234.56, 123.456, 12.3456, 1.23456, .123456.

SQUARE ROOT.

480. The root of a number is one of the equal factors which produced it.

481. The Square Root of any number is one of the

two equal factors which produced it, and may be indicated by the radical sign; thus: $\sqrt{25}$. Thus, the square root of 25 is 5; of 36 is 6; of 49 is 7; of 100 is 10; of 225 is 15; of 324 is 18.

For,

1st. 5×5 produces 25

2d. 6×6 produces 36.

3d. 7×7 produces 49.

4th. 10×10 produces 100.

5th. 15×15 produces 225.

6th. 18×18 produces 324.

482. The finding, or extracting the square root of a number consists in resolving that number into *two equal factors*.

483. In a few cases, as those given, this may be effected by inspection; but in most cases this cannot be so easily effected.

484. To extract the square root of a number.

1. What is the square root of 625?

FIRST METHOD.

Resolving 625 into its prime factors, we find them to be 5, 5, 5, and 5. Hence, as the power is the product of *two equal factors*, one of these factors must be 5×5 , or 25. Therefore, 25 is the square root of 625.

2. What is the square root of 1296?

Suggestion. — What are the prime factors of 1296?

What is the product of *one of each couple* of the equal factors?

3. What is the square root of 5625?

4. What is the square root of 55696?

5. What is the square root of 1679616?

485. The square root of those numbers only which are perfect squares can be found by this method.

SECOND METHOD.

486. To determine the relation of a number to its square.

1. What is the square of 25?

Explanation.—The number 25 consists of 20 *tens* plus 5 *units*. Then, the square of 25 is the product of 20 *tens* plus 5 *units* multiplied by 20 *tens* plus 5 *units*.

First, 20 times 20 gives 400, $(20)^2$

Second, 20 times 5 gives 100, or 20×5 .

Third, 5 times 20 gives 100, or 20×5 .

Fourth, 5 times 5 gives 25, or $(5)^2$

OPERATION.		
Tens.		Units.
20	+	5
20	+	5
$(20)^2$	+	20×5
		$20 \times 5 + (5)^2$

The sum of these several partial products

gives as the total product..... $(20)^2 + 2 (20 \times 5) + (5)^2$

487. Hence, *The square of any number consisting of tens and units is composed of the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

488. From this general law it follows

- That if from the *square* of a number, the *square* of the *tens* figure in the root be subtracted, the remainder will consist,
- Of *twice* the *tens* figure increased by the *units* figure, all multiplied by the *units* figure.

Hence,

- c. The units figure may be found by dividing this remainder by *twice* the *tens* figure of the root.

489. Since every number may be regarded as composed of tens and units, the above law applies to every number, and the deductions marked *a*, *b*, *c*, apply to number in general.

490. From this analysis we derive the following rule for extracting the square root of any number.

GENERAL RULE.

I. *To determine the number of figures in the root, place a dot over units, hundreds, ten-thousands, and so on.*

II. *Find the greatest square in the left period, and place its root at the right, as in division.*

III. *Subtract the square of the root already found from the left period, and bring down the next period.*

IV. *Double the root already found, and place it at the left for a trial divisor.*

V. *Write the figure expressing the number of times the trial divisor is contained in the dividend (exclusive of its right-hand figure), as the second figure of the root.*

VI. *Annex the figure of the root to the trial divisor, and multiply this result by the last figure of the root.*

VII. *Subtract this product from the dividend, and bring down the next period.*

VIII. *Double the root already found, and proceed as before until the last period is exhausted. The root thus found will be the root required.*

NOTES. — 1. When the given number is not a perfect square its root will be expressed by a whole number and decimal.

2. In determining the number of figures in the decimal part of the root, place a dot over hundredths, ten-thousandths, millionths, and so on.

3. When the trial divisor is not contained in the dividend place a cipher in the root, and proceed as before.

4. If the *remainder* in any instance should be greater than the trial divisor, the last quotient figure may have been taken too small, and it may *not*. This can be determined only by trial.

5. The square root of an imperfect square can be expressed only approximately. The operation should be carried, generally, to six decimals in the root.

6. When either term of a fraction is not a perfect square, reduce the fraction to a decimal and then extract its root.

7. Sometimes, by reducing a fraction to lower terms it may be rendered a perfect square.

1. What is the square root of 67305616?

Explanation. — By placing a dot over the alternate figures we find the root will contain *four* figures.

The greatest square in the left period, 67, is 64. The square root of 64 is 8.

The first dividend is 330. The trial divisor is contained in 330, exclusive of the right hand figure, 2 times.

Annexing 2 to the trial divisor, the result, or complete divisor is 162; multiplying this by 2, subtracting and bringing down the next period, 56, the dividend is 656.

The second trial divisor is not contained in the dividend, so we place a cipher in the root, and bring down the next period, 16. This makes the new dividend, 65616.

The third trial divisor, which is 1640, is contained in 65616, exclusive of the right hand figure, *four* times.

Annexing this figure to the root, and multiplying the complete

divisor thus formed, by 4, there is no remainder.

Hence, the square root of 67305616 is 8204.

OPERATION.

			67305616 (8204 root.
		64	
1st trial divisor.....	2×8	$= 16$	) 330
Complete divisor...		162	324
2d trial divisor.....	2×82	$= 164$	) 65616
3d trial divisor.....	2×820	$= 1640$	
3d complete divisor		16404	65616
			00000 Final remainder.

NOTE. — Had there been a remainder after subtracting 65616, the operation could have been continued by annexing *two ciphers* after each division, and proceeding as before, the figures of the root, thus obtained, being decimals.

Extract the square root of

1. 4.	11. 132	21. 484.	
2. 9.	12. 144.	22. 441.	
3. 16.	13. 169.	23. 576.	
4. 25.	14. 196.	24. 529.	
5. 36.	15. 225.	25. 625.	
6. 49.	16. 256.	26. 729.	
7. 64.	17. 289.	27. 676.	
8. 81.	18. 324.	28. 961.	Ans. 31.
9. 100.	19. 361.	29. 1681.	Ans. 41.
10. 121.	20. 400.	30. 2601.	Ans. 51.

31. Find the square root of 12345.

32. Find the square root of 234567.

33. What is the square root of 3456789?

34. What is the square root of 234567890?

35. What is the square root of 987654321?

491. To extract the square root of a common fraction.

1. Extract the square root of $\frac{1}{4}$.

Suggestion. — What fraction multiplied by itself will give $\frac{1}{4}$?
Then, what is the square root of $\frac{1}{4}$?

2. Extract the square root of $\frac{4}{9}$; of $\frac{9}{16}$; of $\frac{16}{81}$.

3. Extract the square root of $\frac{25}{36}$; of $\frac{36}{49}$; of $\frac{49}{81}$.

4. Extract the square root of $\frac{8}{18}$; of $\frac{50}{82}$; of $\frac{13}{1862}$.

5. Extract the square root of $\frac{98}{72}$; of $\frac{242}{72}$.

6. Extract the square root of $1\frac{1}{2}$, $2\frac{1}{8}$, $3\frac{1}{4}$, $4\frac{2}{3}$, $5\frac{4}{5}$.

GEOMETRICAL METHOD.

492. The square of any number is the same as the product of one side of a square by another side.

493. To find the side of a square when its area is given.

1. The area of a square field is 9704 square rods. What is the length of one side of the square?

Suggestion. — The area of a square is found by multiplying its length by its breadth. Since its length equals its breadth, its area equals the square of one of its sides. Hence, to find the side of a square whose area is known, we must resolve the number expressing its area into *two equal factors*.

By carefully examining the annexed diagram, we find that the area of a square is composed of three distinct parts: 1st. The square A. 2d. The square B. 3d. The two equal oblongs represented by C.

Now, the side of the square A may be expressed by the *tens* figure of the root required.

The side of the square B may be represented by the *units* figure of the root.

The area of the square A is expressed by the square of the *tens*

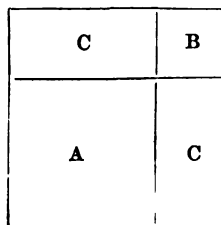


figure of the root. The area of the square B is expressed by the square of the units figure of the root.

The area of each of the oblongs marked C, is expressed by the product of the *tens* by the *units* figure of the root. And the area of both the oblongs is expressed by *twice* the product of the *tens* by the *units* figure of the root.

Hence, the area of the whole square composed of the several parts A, B, and C, equals *the square of the tens plus twice the product of the tens by the units, plus the square of the units*.

The square root of 9704, found by the rule, is 98. Hence, the square A equals $(90)^2$; the square B equals $(8)^2$. The two oblongs represented by C equal $2(90 + 8)$.

Finally, since the area of the original square is 9704 square rods, and since its area is the square of one of its sides, and since the square root of 9704 is 98, then the side of the square is 98 rods in length.

494. A variety of problems, more or less difficult, may be solved by an application of the principles involved in square root.

1. Find the side of a square whose area shall be four times as great as one, the side of which is 41 feet.

Suggestion. — What is the area of a square 41 feet on a side?
What is four times its area?
What is the side of a square containing this area?

2. Find the side of a square having the same area as an oblong 64 feet by 16 feet?

Suggestion. — What is the area of the given oblong?
What is the side of a square equal in area?

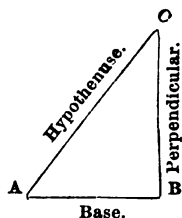
3. An oblong containing 10368 square feet, is twice as long as it is wide. How long, and how wide is it?

Suggestion. — If it was just as long as it is wide, what would be its length? What its shape?
What would be its area?
What the length of a side of this figure?
This dimension corresponds to what dimension of the oblong?

445. A triangle is a plane figure bounded by three straight lines called its sides.

496. A right-angled triangle is a triangle having two of its sides perpendicular to each other, as in the figure in the margin.

497. The sides of a right-angled triangle have special names. The sides enclosing the right angle, as AB , BC , are the base and perpendicular, respectively. The side lying opposite the right angle is the hypotenuse.



498. These three sides bear a constant relation to each other, such that,

The square of the hypotenuse equals the sum of the squares of the base and perpendicular. Hence,

499. Any two sides of a triangle being known, the third side may be found as follows :

1. To find the hypotenuse when the base and perpendicular are known :

Add the square of the base to the square of the perpendicular, and extract the square root of their sum.

2. To find the base when the hypotenuse and perpendicular are known :

From the square of the hypotenuse subtract the square of the perpendicular, and extract the square root of the difference.

3. To find the perpendicular when the hypotenuse and base are known :

From the square of the hypotenuse subtract the square of the base, and extract the square root of the difference.

EXAMPLES.

1. In a right-angled triangle the base is three feet, and the perpendicular 4 feet. What is the length of the hypotenuse? Ans. 5 feet.

2. In a right-angled triangle the base is 24 feet, and the perpendicular 32 feet. What is the hypotenuse? Ans. 40 feet.

3. In a right-angled triangle the base is 7, and the perpendicular 24. What is the hypotenuse? Ans. 25 feet.

4. In a right-angled triangle the base is 13, and the perpendicular 64. What is the length of the hypotenuse? Ans. 65.

5. Find the base of a right-angled triangle whose perpendicular is 40, and whose hypotenuse is 41.

6. Find the base of a right triangle whose perpendicular is 36 feet, and hypotenuse 45 feet.

7. Find the base of a right-angle triangle whose perpendicular is 68, and hypotenuse 85.

8. If the hypotenuse is 181, and the perpendicular is 180, what is the length of the base?

9. A tree standing on a plane was broken by the wind, 20 feet from the ground, and the top of the tree struck the ground 15 feet from the foot of the tree. What was the height of the tree before it was broken?

Suggestion. — What figure is formed by the two parts of the tree and the ground?

What is the length of the base? The perpendicular?

The hypotenuse? The tree?

10. A public park, in the form of a square, is 81 rods on a side. What is the distance between its opposite corners?

11. A room is 18 feet wide, and 24 feet in length. What is the distance on the floor between its opposite corners?

12. If the height of the above room is 15 feet, what is the distance from one of its corners on the floor to the opposite upper corner?

13. The distance between two trees standing on a plane, measured on the ground, is 200 feet; one of the trees is 150 feet high, and the other 100 feet high. Find the distance between their tops.

14. The area of a square is 6241 square rods. Find the distance between its opposite corners.

15. The edge of a certain cube is 21 inches. Find the greatest distance between two of its corners.

16. In the last example, what is the perpendicular distance from the center of the cube to one of its faces?

17. Two men start, at the same time, from the same corner of a rectangular park, and walk with equal speed until they meet. One of them walks diagonally across the park, and turns to meet the other; the other walks along the sides of the park. The park is twice as long

as it is wide, and contains 256 square rods. How many feet along the side of the park must the first man travel?

18. How many feet must the side of a square 80 feet long, be increased, to contain five times as many square feet?

19. Find the side of a square which shall contain one-third as many square rods as one whose side is 18 rods in length.

20. A steeple, partly finished, cast a shadow 92 feet in length, at a certain time of day. When the steeple was completed it cast a shadow 48 feet longer. The distance from the top of the steeple to the edge of its shadow at this time, being 210 feet. What was its height when its shadow was 92 feet?

CUBE ROOT.

500. The **Cube Root** of a number is one of the **three equal factors** which produced it, and may be indicated by the radical sign, thus: $\sqrt[3]{27}$.

501. The square root is indicated by $\sqrt{}$, or $\sqrt[2]{}$; but to indicate any other root an appropriate index must be used in connection with the radical sign. Thus: the cube root of 27 is 3; of 64 is 4; of 125 is 5; of 8 is 2; of 216 is 6. For,

1st $3 \times 3 \times 3$ produces 27.

2d. $4 \times 4 \times 4$ produces 64.

3d. $5 \times 5 \times 5$ produces 125.

4th. $2 \times 2 \times 2$ produces 8.

5th. $6 \times 6 \times 6$ produces 216.

502. Finding the cube root of a number, then, consists in resolving that number into **three equal factors**.

503. In the few cases given this may be done by simple inspection, but in most instances the process is somewhat laborious.

504. To extract the cube root of a number.

FIRST METHOD.

1. What is the cube root of 1728?

Suggestion. — What are the prime factors of 1728?

What is the product of *one of each three* of the equal factors?

2. What is the cube root of 59319? Ans. 39.
3. What is the cube root of 15625?
4. What is the cube root of 592704? Ans. 84.
5. What is the cube root of 1331?
6. Find the cube root of 54872.
7. Find the cube root of 59319.
8. Find the cube root of 68921.
9. Find the cube root of 64000.
10. Find the cube root of 166375.
11. Find the cube root of 91125.
12. Find the cube root of 421875.
13. Find the cube root of 512000.
14. Find the cube root of 729000.
15. Find the cube root of 778688.

505. The cube root of those numbers only which are **perfect cubes** can be found by this method.

SECOND METHOD.

506. To determine the relation of a number to its cube.

1. What is the cube of 24?

Explanation.—The number 24 consists of 20 *tens* plus 4 *units*. Then, the cube of 24 is the product of 20 *tens* plus 4 *units*, taken *three* times as a factor.

First. 20 times 20 gives
400, or $(20)^2$.

Second. 20 times 4 gives
80, or 20×4

Third. 4 times 20 gives
80, or 4×20

Fourth. 4 times 4 gives
16, or 4×4

Hence, the square of 20 +
4, or $(20 + 4)$ $(20 + 4)$, is...

Fifth. In a similar manner, by multiplying the product thus obtained, by the third factor $(20 + 4)$, we find the cube of $(20 + 4)$ to be.....

This may be put under the form

OPERATION.

tens. *units.*

20×4 1st. factor.

20×4 2d. factor.

$(20)^2 + 20 \times 4$

$20 \times 4 + (4)^2$

$(20)^2 + 2(20 \times 4) + (4)^2$

$20 + 4$, 3d. factor.

$(20)^3 + 2[(20)^2 \times 4] + 20 \times (4)^2$

$(20)^2 \times 4 + 2[(20 \times (4)^2)] + (4)^3$

be..... $(20)^3 + 3(20)^2 \times 4 + 3(20 \times (4)^2) + (4)^3$. a.

This may be put under the form $(20)^3 + \{ 3(20)^2 + 3(20 \times 4) + 4^2 \} 4$. b.

507. Hence, *The cube of any number consisting of tens and units is composed of the cube of the tens, plus three times the square of the tens, multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

508. Thus, the cube of any number consists of **four** parts expressed in terms of the *tens* and *units* composing it.

1st. Part. The cube of the tens.

2d. Part. Three times the square of the tens, multiplied by the units.

3d. Part. Three times the tens, multiplied by the square of the units.

4th. Part. The cube of the units.

509. From this general law it follows :

a. That, if from the *cube* of a number, the *cube* of the *tens* figure in the root be subtracted, the remainder will consist,

b. Of two factors, one of which is the units figure of the root.

c. And the other factor is composed of the sum of

1. Three times the square of the tens figure of the root.

2. Three times the tens figure, multiplied by the units figure.

3. The square of the tens figure of the root.

510. One of these factors we obtain in the form of a trial divisor, as in extracting the square root, by taking the results indicated in 1, 2, and 3.

511. Since every number may be regarded as consisting of tens and units, the above law applies to every number, and the deductions marked *a*, *b*, *c*, apply to number in general.

512. From this analysis we derive the following rule for extracting the cube root of any number :

GENERAL RULE.

I. To determine the number of figures in the root, place a dot over units, thousands, millions, and so on.

II. *Find the greatest cube in the left period, and place its root at the right, as in division.*

III. *Subtract the cube of the root already found from the left period, and bring down the next period.*

IV. *Square the part of the root found and multiply by three, for a trial divisor.*

V. *Write the figure expressing the number of times the trial divisor is contained in the dividend (exclusive of its two right-hand figures), as the second figure of the root.*

VI. *Multiply the last figure of the root already found by the rest of the root, and by three, and annex this product to the trial divisor. Complete the divisor by annexing the square of the last figure of the root.*

VII. *Multiply the complete divisor thus found, by the last figure of the root, subtract the product from the dividend, and bring down the next period.*

VIII. *Square the root now found and multiply by three for a trial divisor. Complete the trial divisor as before, multiply by the last figure of the root, subtract, and bring down the next period.*

IX. *Proceed in the same manner until all the periods are brought down.*

NOTES. — 1. If, after the last period is brought down, there is a remainder, annex ciphers for the next period, and proceed as before. The figures of the root thus found will be the decimal part of the root.

2. In determining the number of decimal figures in the root, point from units place toward the right, placing a dot over every third figure.

3. When the trial divisor is not contained in the dividend, place a cipher in the root, and proceed as before.

4. The cube root of a fraction, when its terms are perfect, may be found by extracting the cube root of both of its terms.

5. A fraction may sometimes be made a perfect cube by reducing it to lower terms.

6. If a fraction when reduced to lower terms is not a perfect cube, change it to a decimal, and then extract its cube root.

1. What is the cube root of 1728?

Explanation.

The root will contain two figures. The greatest cube in the left period, 1, is 1; its cube root is 1.

Cubing and subtracting, the first new dividend is 728. The first trial divisor is

1 times 1 times 3,

which is 3. The additions to be made to the trial divisor are, 1st. 1 times 2 times 3, which is 6. 2d. 2 times 2, which is 4.

The complete divisor, then, is 364. Finally, the product of 364 by the last figure of the root, which is 2, gives 728, which exhausts the dividend. Hence, the cube root of 1728 is 12.

OPERATION.

1728(12. Root.

$$1 \times 1 \times 1 = \dots\dots\dots 1$$

1st. Trial Divisor —

$$\text{is } (1)^2 \times 3 = 3) 728 \quad \text{1st. New Dividend.}$$

$$1 \times 2 \times 3 = \dots\dots 6$$

$$2 \times 2 = \dots\dots 4$$

Complete Di. —

$$\text{visor} \dots\dots\dots 364 \ 728$$

000 Remainder.

Suggestion.—Let the pupils extract the cube root of the examples under Art. 504, by this method.

Extract the cube root of the following fractions :

1. $\frac{1}{8}$.

7. $\frac{54}{128}$.

13. .2.

2. $\frac{1}{27}$.

8. $\frac{16}{128}$.

14. .03.

3. $\frac{1}{64}$.

9. $\frac{128}{2500}$.

15. .0356.

4. $\frac{8}{27}$.

10. $\frac{24}{81}$.

16. .4126.

5. $\frac{8}{48}$.

11. $\frac{248}{24}$.

17. .56816.

6. $\frac{27}{64}$.

12. $\frac{482}{8100}$.

18. .123456.

EXAMPLES.

1. Find the side of a cube which contains 8 times as many cubic feet as a cube whose edge is $\frac{1}{7}$ of a foot.

2. Find the edge of a cube which shall contain the same number of cubic inches as a solid 216 inches long, 27 inches wide, 8 inches thick.

3. How large a cubic bin will contain as many bushels as a bin 20 feet long, 7 feet wide, and 5 feet deep?

4. How many small cubes can be cut from a large cube, allowing nothing for waste, the edge of each small cube being $\frac{1}{4}$ of an inch, and that of the larger cube being 2 feet? Ans. 884736.

NOTE. — For further exercises in the application of cube root, see the subject of Analysis, Art. 579.

ARITHMETICAL PROGRESSION.

513. An **Arithmetical Progression** is a series of numbers which increases or decreases by a **common difference**; as,

First: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.	}	Ascending Series.
3, 5, 7, 9, 11, 13, 15, 17.		
2, 5, 8, 11, 14, 17, 20, 23.		
Second: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.	}	Descending Series.
17, 15, 13, 11, 9, 7, 5, 3,		
23, 20, 17, 14, 11, 8, 5, 2.		

514. When each succeeding term increases by a **common difference**, the series is an **ascending series**; as, the three series marked First, in art. 513.

515. When each succeeding term decreases by a **common difference** the series is a **decreasing series**; as, the three series marked Second, above.

516. In any arithmetical series, whether ascending or descending, only **five** quantities are considered, viz. :

1st. The **First term**.

2d. The **Common difference**.

3d. The **Number** of terms.

4th. The **Last term**.

5th. The **Sum** of the series.

517. Any **three** of these quantities being known, the other two quantities can be found.

518. These five quantities may be combined so as to give rise to **twenty** different problems.

519. **Three** of these five quantities must be **known** before either of the other two can be determined.

520. Only **four** of the twenty possible problems are usually considered in arithmetic.

CASE I.

521. To find the last term when the first term, the common difference, and the number of terms are known.

Example 1. — Find the last term in the series, the first term of which is 3, common difference 2, and number of terms 7.

This series may be written as follows :

1st Term.	2d Term.	3rd Term.	4th Term.	5th Term.	6th Term.	7th Term.
3	5	7	9	11	13	15

Suggestion. — An examination of this series discloses the following facts :

- a. The **second** term is found by adding the common difference **once** to the first term.
- b. The **third** term is found by adding the common difference **twice** to the first term.
- c. The **fourth** term is found by adding the common difference **three** times to the first term.
- d. The **fifth** term is found by adding the common difference **four** times to the first term.
- e. The **sixth** term is found by adding the common difference **five** times to the first term.
- f. Finally, the **seventh** term is found by adding the common difference **six** times to the first term.

522. From this analysis we derive the following rule for finding the last term, when the first term, common difference, and number of terms are known :

RULE. — *Multiply the common difference by the number of terms less one, and add the product to the first term, if an increasing series; but subtract it from the first term, if a decreasing series. The result will be the last term.*

Example 2. — Find the last term in the series 5, 9, 13, to ten terms.

3. The first of an arithmetical progression is 7, the common difference is 4, and the number of terms is 21. Find the last term.

Ans. 87.

4. Find the last term in the series 100, 95, 80, continued to ten terms. Ans. 55.

CASE II.

523. To find the number of terms when the first term, the last term, and the common difference are known.

Example 1. — In a series the first term is 7, the last term is 47, and the common difference is 4. Find the number of terms.

- Suggestion.** — *a.* How is the *last* term found? Case I.
b. The difference between the first and last terms equals what?
c. This difference is the product of what two factors?
d. Which of these two factors is known?
e. How is the other factor found?
f. Then, what is the number of terms?

Full Analysis. — The last term is the sum of two quantities. The first term is one of these two quantities.

The other quantity is the product of two factors.

The common difference is one of these two factors.

The other factor is a number one less than the number of terms.

524. From this analysis we derive the following rule for finding the number of terms, when the first and last terms and the common difference are known.

RULE. — *From the last term subtract the first term, divide the remainder by the common difference, and add 1 to the quotient. The result will be the number of terms.*

CASE III.

525. To find the common difference when the first term, the last term, and the number of terms are known.

Example 1. — The first and last terms of an arithmetical series are 1 and 45, and the number of terms 23. What is the common difference?

- Suggestion.** — *a.* How is the last term found? Case I.
b. The difference between the first and last terms equals what?
c. Which factor in this product is known?
d. How is the other factor found?
e. Then, what is the common difference?

Fuller Analysis. — The last term is the sum of what two quantities?

Which of these two quantities is known? Both?

Their difference is the product of what two factors?

The number of terms, less 1, is one of these two factors.

The common difference is the other.

526. From this analysis we derive the following rule for finding the common difference, when the first term, last term, and the number of terms are known.

RULE. — *Divide the difference between the first and last terms by the number of terms, less 1. The quotient will be the common difference.*

2. Find the common difference in the series whose first and last terms are 8 and 203, and the number of terms 40.

3. The youngest of 11 children is 4, and the oldest is 44. The difference between their ages is the same. Find that difference.

CASE IV.

527. To find the sum of the series —

a. When the first, last, and number of terms are known.

b. When the first term, common difference, and number of terms are known.

Example 1. — The first term is 6, and last term 42, and the number of terms 10. Find the sum of the series.

Suggestion. — To construct this series we must find the common difference. This is found, by Case III., to be 4.

The series is 6, 10, 14, 18, 22, 26, 30, 34, 38, 42.

Reversed, 42, 38, 34, 30, 26, 22, 18, 14, 10, 6.

Sum, 48, 48, 48, 48, 48, 48, 48, 48, 48, 48.

Thus we see that 48, the sum of the first and last terms, when taken *ten* times, that is, as many times as there are terms in the series, gives *twice* the sum of the series.

Hence, 480 is twice the series; or, 240 is the sum of the series.

528. From this we derive the following rule for finding the sum of an arithmetical series, when the first and last terms, and the number of terms are known.

RULE. — *Multiply the sum of the first and last terms by half the number of terms.*

NOTE. — When the first term, the number of terms, and the common difference are known, the *last* term must be found before applying the rule.

2. How much would a man save in 21 years, if he should save \$100 the first year, \$150 the second, \$200 third, and so on? Ans. \$12600.

GEOMETRICAL PROGRESSION.

529. A Geometrical Progression is a series of numbers which increases or decreases by a common multiplier, called the ratio, as —

First: 1, 2, 4, 8, 16, 32, 64, 128. }
 3, 9, 27, 81, 243, 729, 2187. } Ascending Series.
 5, 20, 80, 320, 1280, 5120, 20480. }

Second: 243, 81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$. }
 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$. } Descending Series.
 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$. }

530. When each succeeding term increases by a common multiplier the series is ascending.

531. When each succeeding term decreases by a common multiplier the series is descending.

532. When the series is descending the ratio is a proper fraction.

533. In any geometrical series only five quantities are considered, viz. :

- 1st. The first term.
- 2d. The ratio.
- 3d. The number of terms.
- 4th. The last term.
- 5th. The sum of the series.

534. Any three of these quantities being known, the other two quantities can be found.

535. These five quantities may be combined so as to give rise to twenty different problems.

536. Three of these five quantities must be **known** before either of the other two can be determined.

537. Only **four** of these twenty possible problems are usually considered in arithmetic.

CASE I.

538. To find the last term, when the first term, the number of terms, and the ratio are known.

Example 1. — The first term of a geometrical series is 3, the number of terms 6, and the ratio 2. Find the last term.

This series may be written as follows :

1st Term.	2d Term.	3d Term.	4th Term.	5th Term.	6th Term.
3.	6.	12.	24.	48.	96.

Suggestion. — An examination of this series discloses the following facts :

- a. The *second* term equals the *first* multiplied by the *ratio*.
- b. The *third* term equals the *first* multiplied by the *ratio squared*.
- c. The *fourth* term equals the *first* multiplied by the *ratio cubed*.
- d. The *fifth* term equals the *first* multiplied by the *fourth* power of the *ratio*.
- e. The *sixth* term equals the *first* multiplied by the *fifth* power of the *ratio*.

GENERAL LAW OF THE SERIES.

539. *Any term in a geometrical series equals the first term, multiplied by the ratio raised to a power whose index is one less than the number of the term.*

Find the last term in the following series :

2. First term 1, ratio 4, number of terms 8.
3. First term 2, ratio 2, number of terms 10.
4. First term 3, ratio 3, number of terms 6.
5. First term 1000, ratio $\frac{1}{5}$, number of terms 4.
6. First term 7500, ratio $\frac{1}{10}$, number of terms 3.
7. First term 512, ratio $\frac{1}{2}$, number of terms 12.
8. First term $\frac{1}{2}$, ratio $\frac{1}{2}$, number of terms 5.

CASE II.

540. To find the number of terms, when the first term, the last term, and the ratio are known.

Example 1. The first term of a geometrical series is 3, and the last term is 2187; find the number of terms, the ratio being 3.

Suggestion. — *a.* How is the last term found? Case I.

b. The last term divided by the first gives what power of the ratio?

c. The number of terms is one greater than the index of this power.

Fuller Analysis. — The last term, 2187, divided by the first term, 3, gives 729. Now, 729 is the 6th power of 3; hence, the number of terms must be 7.

In a similar manner find the number of terms in the following examples :

2. First term 1, last term 8, ratio 2.
3. First term 1, last term 16, ratio 2.
4. First term 1, last term 64, ratio 2.
5. First term 3, last term 81, ratio 3.
6. First term 3, last term 243, ratio 3.
7. First term 5, last term 320, ratio 4.
8. First term 5, last term 1280, ratio 4.

CASE III.

541. To find the ratio, when the first and last terms, and the number of terms are known.

Example 1. — In a geometrical series the first and last terms are 3 and 729, and the number of terms 6; what is the ratio?

Suggestion. — *a.* How is the last term found? Case I.

b. The last divided by the first term gives what power of the ratio?

c. The index of this power is indicated by the number of terms, less one.

Fuller Analysis. — The first term, 3, is contained in the last term, 729, just 243 times. The number of terms, less one, is 5; hence, 243 is the fifth power of the ratio. The fifth root of 243 is 3, the ratio.

Find the ratio in the following examples :

2. First and last terms are 1 and 16, and the number of terms 5.

3. First and last terms are 1 and 32, and the number of terms 6.

4. First and last terms are 3 and 243, and the number of terms 5.

5. First and last terms are 5 and 1280, and the number of terms 5.

CASE IV.

542. To find the sum of the series, when the first and last terms, and the ratio are known.

Example 1. — The first and last term of a geometrical series are 3 and 243, and the ratio 3. Find the sum of the series.

Once the Series	3, 9, 27, 81, 243.	} Taking the difference to get twice the series.
Three times the Series	9, 27, 81, 243, 729.	
Twice the Series	729—3, or 726.	

Hence, $\frac{1}{2}$ of 726, or 363, must be the sum of the first series.

543. From this explanation we derive the following rule :

RULE. — *Multiply the greatest term in the series by the ratio, from this product subtract the least term, and divide the remainder by the ratio diminished by one.*

Find the sum of the series when—

2. The first and last terms are 1 and 128, and the ratio 2.

3. The first and last terms are 3 and 2187, and the ratio 3.

4. The first and last terms are 5 and 20480, and the ratio 4.

NOTE a. — When the series is descending, instead of dividing by the ratio diminished by one, we must divide by 1 diminished by the ratio; since the ratio is a proper fraction.

NOTE b. — The subject of Progressions belongs properly to algebra, where it will be found treated with great fullness.

INFINITE DECREASING SERIES.

544. To find the sum of an infinite decreasing series when the first term and the ratio are known.

Example 1. — Find the sum of $\frac{1}{8} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$, and so on to an infinite number of terms.

Explanation. — Multiplying this series by 3, we have —

a. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \&c.$, to infinity. Resulting Series.

b. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \&c.$, to infinity. Original Series.

Hence, subtracting the series marked *b* from the series marked *a*, we get 1 as the difference between the two series. But, *three* times the series minus *once* the series gives a result which is *twice* the series. Hence, 1, the difference, is *twice* the sum of the series; that is, $\frac{1}{2}$ is the sum of the given series.

NOTE. — Since the ratio is $\frac{1}{3}$ in this case, the multiplier is 3. When the ratio is $\frac{1}{4}$ the multiplier must be 4; if $\frac{1}{5}$, 5. Generally, the multiplier must be the *reciprocal* of the ratio.

545. The reciprocal of any number is 1 divided by that number.

546. The ordinary method of summing an infinite decreasing geometrical series by regarding its *last* term as zero, is grossly absurd. A series having an infinite number of terms has no *last* term.

MISCELLANEOUS PROBLEMS.

1. If a man should travel 13 days, increasing his rate of travel 5 miles each day, how far must he go the first day, and how far altogether, if, on the last day of his journey, he should travel 80 miles?

2. If a father should allow his son one cent on the first day of the month, 2 cents the second day, 4 cents the third, 8 cents the fourth, and so on for every day in the month of June, what would be his allowance for the entire month?

3. A man hired a livery horse to ride 20 miles, agreeing to pay 1 mill for the first mile, and twice as much for each mile as for the preceding mile. How much did the ride cost him?

4. Find the sum of the series $\frac{2}{8}, \frac{1}{8}, \frac{1}{24}, \frac{1}{96}$, and so on to infinity.

Ans. $\frac{5}{8}$.

5. Find the sum of the series $1\frac{1}{2}, \frac{1}{2}, \frac{1}{8}$, and so on to infinity.

Ans. $\frac{9}{4}$.

6. Find the sum of the infinite series $3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}$, and so on.

Ans. 4.

7. A body moving through space at the rate of $\frac{1}{8}$ of a foot in the first second of time, only $\frac{1}{8}$ of a foot the second second of time, $\frac{1}{27}$ of a foot the third second,

would move over what entire distance, provided it should never stop moving? $\frac{1}{2}$ a foot.

8. The first term of an arithmetical series is 4, the number of terms is 17, the common difference is 3. Find the last term. Ans. 52.

9. Find the last term of the arithmetical series 1, 3, 5, 7, and so on, the series consisting of 23 terms. Ans. 45.

10. Find the last term in the series 29, 27, 25, the number of terms being 11. Ans. 9.

11. Find the number of terms in the series 7, 10, 13, 16, the last term being 31. Ans. 9.

12. Find the number of terms in the series 34, 31, 28, the last term being 4. Ans. 11.

13. The last term of an arithmetical series is 37, the number of terms 10, and the first term 7. Find the common difference.

14. The last term of a series is 1, the first term 41, and the number of terms 13. What is the common difference?

15. One hundred silver dollars were placed in a straight line on the ground, at the distance of ten feet from each other. If the first one was three feet from a basket in which they were to be deposited, how far must a person walk to gather them up singly and deposit each in the basket, and return for the next, if he should start from the basket?

16. Wishing to ascertain the height of a certain tower, I dropped a marble from its summit, and found it reached the ground in 9 seconds of time. Now, if

a body falls, by its own weight, $16\frac{1}{2}$ feet the first second of time, 3 three times that distance the second second of time, 5 times that distance the third second, and so on, what was the height of the tower?

MENSURATION.

547. A plane figure is a surface bounded by either straight or curved lines.

548. A square is a plane figure bounded by four equal straight lines, and having four equal angles.

549. A rectangle is a plane figure having its opposite sides equal straight lines, and all its four angles equal.

550. A parallelogram is a plane figure having its opposite sides equal and parallel; but, of its four angles, only those opposite each other are equal.

NOTE.—The pupil should draw each of these figures.

551. To find the area of a square, rectangle, or parallelogram.

RULE. — *Multiply its length by its breadth.*

Example 1. — Find the area of a square whose side is 25 feet.

2. Find the area of a square whose side is 42 feet.

3. A square field is 320 rods on a side; what number of acres does it contain?

4. A farmer has a field in the form of a rectangle; what number of acres does it contain, if its length is 76 rods, and width 34 rods?

5. A field in the form of a rectangle is 25 rods long, and the distance between its opposite corners is 313 rods. Find the contents of the field, in acres.

552. To find the are of a triangle.

A triangle is a plane surface enclosed by three straight lines, and is equal in area to one-half a square or rectangle having the same base and altitude. Hence, the—

RULE.—a. *Multiply one-half its base by its altitude; or,*

b. *Multiply one-half its altitude by its base.*

1. Find the area of a triangle whose base is 15 feet and height 12.

2. The base of a triangle is 47, and its altitude 27. Find its area.

553. To find the area of a triangle when the length of its three sides is known.

RULE.—*Find half the sum of its three sides. From this half sum subtract each of the sides. Find the product of this half sum and these three remainders, and extract the square root.*

1. Find the area of a triangle whose sides are 20, 30, and 40.

2. Find the area of a triangle whose sides are 12, 18, and 24.

3. Find the area of a triangle whose sides are 50, 75, and 100.

554. To find the area of a trapezoid.

A trapezoid has two of its four sides parallel, but unequal.

RULE. — *Multiply one-half the sum of its parallel sides by its altitude.*

1. In a trapezoid the parallel sides are 22 and 34, and its altitude 16. What is its area?

2. Find the area of a trapezoid whose parallel sides are 18 and 26, and altitude 12.

3. How many square feet in a board 16 feet long, one end of which is 17 inches wide and the other only 11 inches wide?

4. Find the area of a cube 12 inches on its edge.

Ans. 6 sq. ft.

5. Find the superficial contents of a timber 11 feet long, 13 inches wide, and 12 inches thick.

Ans. 48 sq. ft.

6. How many feet of lumber are required to make a grain-bin 8 feet long, 5 feet wide, and 4 feet deep?

Suggestion. — How many sq. ft. in the two ends?

How many sq. ft. in the two sides?

How many sq. ft. in the top and bottom?

How many sq. ft. in the entire bin?

7. How many bushels of grain will be required to fill such a bin?

Ans. 128.57 bu.

8. What will it cost to plaster the walls and ceiling of a room which is 14 by 16 feet, and 9 feet high, at 30 cents a square yard?

Ans. \$28.80.

CIRCLES.

555. A circle is a plain figure enclosed by a curved line, every point in which is at the same distance from a point within the circle, called the center.

556. The curve line which encloses the circle is the circumference.

557. The distance from the center to the circumference is the radius.

558. A diameter is a straight line through the center, and limited by the circumference.

559. To find the area of a circle when its radius is known.

RULE. — *Multiply the square of the radius by the constant quantity 3.1416.*

1. The radius of a circle is 48 ; find its area.
2. The radius of a circle is 62 ; find its area.
3. Find the area of a circle whose radius is 91.

560. To find the circumference when the diameter is known.

RULE. — *Multiply the diameter by the constant quantity 3.1416.*

1. The diameter of a circle is 45 ; what is its circumference?
2. Find the circumference, when the diameter is 15.5.
3. Find the circumference of a circle whose radius is 22.1.

561. To find the diameter when the circumference is known.

RULE. — *Divide the circumference by 3.1416.*

1. If the circumference of a circle is 63.2456 ; what is its diameter? What its radius?

2. Find the radius of a circle whose circumference is 281.35.

3. The circumference of a circle is 312.416. Find its radius.

4. How many square feet of sheet-iron are required to make a 6-inch stove-pipe twelve feet in length, and consisting of six joints, allowing one inch for each joint, and one quarter of an inch for the lap?

562. To find the diameter when the area is known.

RULE. — *Divide the area by 3.1416, extract the square root, and multiply by 2.*

1. The area of a circle is 274.36. Find its diameter.

2. What must be the length of a halter, to allow a horse to graze over just one acre?

3. How many more feet of fence are required to enclose just one acre in the form of a square, than are required to enclose just one acre in the form of a circle?

PRISMS.

1. Find the area of a triangular prism 12 inches long, and each of whose sides is 2 inches wide.

2. A prism is 9 inches in length, $2\frac{1}{2}$ inches wide, and $1\frac{1}{2}$ inches thick. How many square inches does it contain?

SOLIDS.

563. To find the solid contents of a prism, or cylinder.

RULE. — *Multiply the area of one of its ends by its length.*

1. A prism that is 4 inches wide and 3 inches thick, is 14 inches long. How many cubic inches in it?

2. A triangular prism whose sides are 16 inches in length and $2\frac{1}{2}$ inches wide, contains how many cubic inches?

564. A cylinder is a solid of uniform diameter, and whose ends are circles which are parallel to each other.

3. The length of a cylinder is 11 feet, and its diameter 15 inches. Find its contents.

4. Find the contents of a cylinder 3 feet long and 3 feet in diameter.

565. To find the surface of a sphere when its diameter is known.

Suggestion. — The area of a sphere equals the area of *four* circles having the same diameter as the sphere.

566. To find the solid contents of a sphere or globe.

1. Find the solid contents of a sphere whose diameter is 3 feet. Find its surface.

Suggestion. — A sphere equals $\frac{2}{3}$ of a cylinder whose diameter and length each equals the diameter of the given sphere.

2. How many cubic inches in a ball of wood 8 inches in diameter.

Ans. 268.08+.

3. Find the cubic contents of a cylindrical tube that will just enclose a sphere whose diameter is 9 inches. Also, find the contents of the sphere.

567. To find the contents of a pyramid.

Suggestion.—A pyramid equals $\frac{1}{3}$ of a cylinder, or prism, having the same base and altitude.

1. The base of a pyramid is a square 5 inches on a side, its height being 21 inches. Find its contents?

Ans. 175 cu. in.

2. Find the contents of a pyramid 10 inches high, with a triangular base 3 inches on a side.

3. Find the *superficial* contents of the above pyramid.

568. To determine the dimensions of a cistern which shall have a given capacity.

1. I wish to dig a cistern which shall have a uniform diameter of five feet, and shall hold 100 barrels. What must be its depth?

	OPERATION.
Cubic inches in a gallon.....	231
No. gallons in one barrel.....	31 $\frac{1}{2}$
	231
	693
	115 $\frac{1}{2}$
Cubic inches in a barrel.....	7276 $\frac{1}{2}$
	100
Cubic inches in 100 barrels.....	727650
Area of base is $(30)^2 \times 3.1416 = 2827.44$ sq. inches.	
$727650 \div 2827.44 = 257.32$ inches for the depth; or, 21.4 ft.	

2. How many barrels will a cistern hold, if its diameter is $4\frac{1}{2}$ feet, and depth $9\frac{1}{2}$ feet?

MECHANICAL POWERS.

569. The **Lever**, the **Wheel and Axle**, the **Screw**, the **Inclined Plane**, and the **Pulley**, are known as the **Five Mechanical Powers**.

570. In the mechanical power known as the lever,

- a. *The fulcrum may be between the power and the weight.*
- b. *The weight may be between the power and the fulcrum.*
- c. *The power may be between the weight and the fulcrum.*

GENERAL LAW.

The power is to the weight as the distance of the power from the fulcrum to the distance of the weight from the fulcrum.

1. In the lever of the first class, what power will raise a weight of 500 pounds at a distance of $2\frac{1}{2}$ feet, the length of the lever being 15 feet? **Ans. 100.**

2. In the lever of the second class, what weight situated $2\frac{1}{2}$ feet from the fulcrum can be raised by a power equal to 100 pounds, by using a lever 15 feet long? **Ans. 600 lbs.**

3. In the lever of the third class, what power will sustain a weight of 50 pounds at the other end of a lever 15 feet long, if the power is applied at a distance of $2\frac{1}{2}$ feet from the fulcrum? **Ans. 300.**

571. In the mechanical power known as the **wheel and axle**, the following principle applies:

GENERAL LAW.

The power is to the weight as the diameter of the wheel is to the diameter of the axle.

4. What weight can a man raise, if he apply a force equal to 100 pounds to a wheel 5 feet in diameter, the axle being 10 inches? Ans. 600 lbs.

5. With a wheel 3 feet in diameter, and axle 9 inches, what power will balance a weight 700 pounds? Ans. 175 lbs.

572. In the mechanical power known as the **inclined plane** the following principle applies :

GENERAL LAW.

The power is to the weight as the perpendicular height of the plane is to the length of the plane.

6. The length of a plane is 20 feet, and its perpendicular height 17 inches. What power will balance a weight of 1000 lbs.?

7. On an inclined plane 16 feet long and 20 inches high, what weight will a force of 100 pounds sustain?

573. In the mechanical power known as the **screw** the following principle applies :

The power is to the weight as the distance between two consecutive threads of the screw to the distance traversed by the extremity of the lever in making one revolution.

8. The distance between the consecutive threads of a screw is one-fourth of an inch, and the length of a

lever 9 feet. What weight can be raised by a force of 75 pounds?

9. What force, with the above screw, will be necessary to raise a weight of 1200 pounds?

10. Find the length of lever necessary to raise 1000 pounds with the same screw, by applying a force of 150 pounds.

574. In the mechanical power known as the **pulley** the following principle applies :

GENERAL LAW.

The power is to the weight to be raised as one to twice the number of movable pulleys.

575. A single stationary pulley serves only the purpose of conveniently applying the power ; it gives no other advantage.

576. With a single pulley, a weight at one end of the rope will just balance a force equal to the same number of pounds at the other end.

577. A single movable pulley doubles the effectiveness of the power.

NOTE.—The consideration of this whole subject falls properly under **Natural Philosophy**. The further consideration of it in this place must necessarily be so meagre as to offer very little profit or pleasure.

In the instances given, the subject of friction, which is a material element when anything more than approximate accuracy is required, is usually left out of the calculation. Other elements, the direction in which the force acts, the nature of the system of pulleys considered, are all too complicated to be discussed here.

578. The following problems have been selected

merely to illustrate the principles enunciated, and not for the purpose of giving the pupil facility in applying them in the solution of difficult problems.

MISCELLANEOUS PROBLEMS.

1. If a man can raise 125 lbs. without the aid of any mechanical contrivance, how many pounds can he move along an inclined plane 44 feet long and 20 feet high?

Ans. 275.

2. In a lever of the first class, 7 feet in length, what weight at one extremity will balance 10 pounds at the other extremity, which is 5 feet from the fulcrum?

Ans. 25.

3. In a system of 5 movable pulleys, what power will sustain a weight of 165 pounds? Ans. 16.5.

4. With what force will a power equal to 30 pounds act, when applied to a lever 3 feet in length, the distance between two contiguous screws being one inch?

Ans. 4523.76 lbs.

5. What weight will a force of 14 pounds sustain, when applied to a wheel 9 feet in diameter, the axle around which the rope attached to the weight is wound being 7 inches? Ans. 216 lbs.

6. At the extremities of a lever 2 weights, 5 pounds and 7 pounds, are placed. If the lever is 24 inches in length, at what point must the fulcrum be placed to produce equilibrium?

Ans. $\left\{ \begin{array}{l} 10 \text{ inches from one.} \\ 14 \text{ inches from other.} \end{array} \right.$

7. At the longer arm of a balance a body is found

to weigh only 9 pounds, but at the shorter arm it is found to weigh 16 pounds. Find its true weight.

Suggestion. — Represent the long arm by a , the short arm by b , and the true weight by x . Then,

$$\text{1st. } a : b :: 16 : x; \text{ or } x = \frac{16b}{a}.$$

$$\text{2d. } b : a :: 9 : x; \text{ or } x = \frac{9a}{b}.$$

Hence, $x^2 = 144$; and $x = 12$.

ANALYSIS.

579. To become proficient in arithmetic, one must be master of arithmetical analysis. A knowledge of special rules is sufficient in special examples falling clearly under these rules. A thorough knowledge of the principles which underlie a subject renders special rules unnecessary. To be able to analyze any new problem implies a command over general principles, and their application in practice, which transcends all special rules. A careful study of the conditions of a problem will usually disclose the method of solution. As a drill in clear and accurate thinking, the practice of solving problems by analysis is invaluable.

The following solutions illustrate a great variety of methods of dealing with the conditions of problems often proposed for solution :

1. If $\frac{1}{3}$ of 6 be 3, what will $\frac{1}{4}$ of 20 be?

Analysis. — If $\frac{1}{3}$ of 6 becomes 3, the whole of 6 becomes 9, and 1 becomes $\frac{1}{3}$ of 9, or $1\frac{1}{3}$. Then 20 becomes 20 times $1\frac{1}{3}$, or 30; $\frac{1}{4}$ of 30 is $7\frac{1}{2}$.

Ans. $7\frac{1}{2}$.

2. John bought 5 apples for 5 cents, and James bought 3 apples for 3 cents. Charles joined them, and the three boys shared the apples equally. If Charles paid the other two 8 cents for his share, how much should each receive?

Analysis. — Since they shared the 8 apples equally, each boy received $\frac{8}{3}$ apples. As James had only $\frac{3}{3}$ apples, he furnished only $\frac{1}{3}$ of an apple for Charles; and as John had $\frac{5}{3}$ apples, he furnished the remaining $\frac{7}{3}$ apples. John should receive 7 times as much as James, or 7 cents, and James should receive 1 cent.

3. A can do a piece of work in 12 days, and B can do it in 15 days. In what time can they both do the work together?

Analysis. — Since A can do it in 12 days, he can do $\frac{1}{12}$ of it in one day. Since B can do it in 15 days, he can do $\frac{1}{15}$ of it in one day. Then they both can do in one day $\frac{1}{12} + \frac{1}{15} = \frac{3}{20}$ of the work. They can do $\frac{1}{20}$ of it in $\frac{1}{3}$ of a day, and $\frac{20}{30}$, or the whole, in $\frac{20}{3} = 6\frac{2}{3}$ days.

Ans. $6\frac{2}{3}$.

4. A, B, and C can do a piece of work in 10 days; A can do it alone in 20 days; B can do it alone in 30 days. In what time can C do the work alone?

Analysis. — A and B together do $\frac{1}{10}$ of the work in a day. They all do $\frac{1}{10}$ of it in 1 day; hence C does $\frac{1}{10} - \frac{1}{20} = \frac{1}{20}$ of the work in 1 day. It will take C 20 days to do the work alone.

5. A, B, and C can do a piece of work in 20 days. A and C can do it in 30 days. In what time can B do it alone?

Analysis. — A and B and C can do $\frac{1}{20}$ of the work in 1 day. A and C do $\frac{1}{30}$ of the work in one day. Then B does $\frac{1}{20} - \frac{1}{30} = \frac{1}{60}$ of the work in one day; hence B can do it alone in 60 days.

6. A and B can do a piece of work in 10 days; A and

C can do a similar piece in 12 days ; B and C can do a similar piece in 15 days. In what time can they all do the work ; and in what time can each do the whole work alone ?

Analysis. — *First.* A and B do $\frac{1}{10}$ of it in 1 day ; A and C do $\frac{1}{12}$ of it in 1 day ; B and C do $\frac{1}{15}$ of it in 1 day. Then they all do $\frac{1}{10} + \frac{1}{12} + \frac{1}{15}$ of the work in 1 day ; that is, $\frac{1}{6}$. Hence they all do the whole work in 6 days.

Second. Since they all do $\frac{1}{6}$, and B and C do $\frac{1}{15}$, A does $\frac{1}{6} - \frac{1}{15} = \frac{1}{10}$ of it in 1 day. Hence A does the whole work alone in $1 \div \frac{1}{10} = 10$ days. In a similar manner B is found to do the work in 24 days, and C in 40 days.

7. A, B, and C do a piece of work together for \$18.60 ; A and B think they can do $\frac{3}{5}$ of it ; A and C think they can do $\frac{2}{3}$ of it ; and B and C think they can do $\frac{4}{5}$ of it. According to these estimates, what should each be paid ?

Analysis. — In this question the estimates exceed the whole work, as the three men, according to their estimates, do $\frac{3}{5} + \frac{2}{3} + \frac{4}{5}$ of the actual work to be done. But as the estimate of A and B is $\frac{3}{5}$ of the work, their estimate is $\frac{3}{5}$ of the actual work done by all ; hence C does $\frac{3}{5} - \frac{3}{5} = 0$ of the work, and should receive $\frac{0}{5}$ of \$18.60, or \$0. A's and C's estimate is $\frac{2}{3}$ of the work ; they do $\frac{2}{3}$ of the actual work done ; hence B does $\frac{2}{3} - \frac{2}{3} = 0$ of the work, and should receive $\frac{0}{3}$ of \$18.60, or \$0. A should receive $\frac{1}{5}$ of \$18.60, or \$3.72.

8. A, B, and C do a piece of work for \$15.40, on the supposition that A and B can do $\frac{3}{5}$ of it, A and C can do $\frac{2}{3}$ of it, B and C can do $\frac{4}{5}$ of it. What should each receive ?

Analysis. — In this question there is an underestimate, as the work done by all is only $\frac{3}{5} + \frac{2}{3} + \frac{4}{5}$ of the whole work. A and B do $\frac{3}{5}$ of the estimated, or $\frac{3}{5}$ of the actual work done. A and C do $\frac{2}{3}$ of the estimated work, or $\frac{2}{3}$ of the actual work done. B and C do $\frac{4}{5}$ of the estimated work, or $\frac{4}{5}$ of the actual work done. Hence, A

should receive $\frac{3}{7}$ of \$15.40, or \$5; B should receive $\frac{4}{7}$ of \$15.40, or \$1; C should receive $\frac{1}{7}$ of \$15.40, or \$9.40.

9. A market woman bought a number of apples at the rate of 2 for a cent, and as many more at the rate of 3 for a cent. She sold both lots at the rate of 3 for 5 cents, and thereby lost 5 cents. How many apples did she purchase?

Analysis. — The first lot was bought at the rate of $\frac{1}{2}$ cent apiece, and the second lot at the rate of $\frac{1}{3}$ cent apiece. Hence, one of each kind, or 2 apples, cost $\frac{5}{6}$ of a cent, or 1 apple cost $\frac{5}{12}$ of a cent. But each apple sold for $\frac{5}{6}$ of a cent; hence, she lost on each apple $\frac{5}{12} - \frac{5}{6} = \frac{1}{6}$ of a cent. To lose 1 cent she must have sold 60 apples, and to lose 5 cents she must have sold 5 times 60, or 300 apples.

10. A lady has two silver cups, but only one cover for both. The first cup is worth \$9; the first cup and cover are worth twice the second cup; and the second cup and cover are worth four times the first cup. What is the value of the second cup, and of the cover?

Analysis. — By the last condition the second cup and cover are worth four times the value of the first cup, or \$36. Then the cover and both cups are worth \$45. By the second condition the cover and both cups equal in value 3 times the second cup. Hence, $\frac{1}{2}$ of \$45, or \$15, is the value of the second cup. The cover is worth \$45 — \$24 = \$21.

11. The head of a fish is 8 inches long. Its tail is as long as its head and half its body, and its body is as long as its head and tail together. How long is the whole fish?

Analysis. — By the second condition its tail is 8 inches plus half the body; hence its head and tail together are 16 inches plus half the body. By the third condition, then, half the body is 16

inches, or the whole body is 32 inches. Hence, the whole length of the fish is 64 inches.

12. At what time between 2 and 3 o'clock are the hands of a watch together?

Analysis.—The minute-hand moves 60 minutes while the hour-hand moves only 5 minutes; or the minute-hand moves 12 minutes while the hour-hand moves 1 minute. Hence, the minute-hand gains 11 minutes in every 12 minutes it moves over. Since the minute-hand moves 12 minutes to gain 11 minutes on the hour-hand, it must move $1\frac{1}{11}$ minutes to gain 1 minute on the hour-hand. To gain any number of minutes on the hour-hand it must move over $1\frac{1}{11}$ of the time to be gained. At 2 o'clock the hour-hand is 10 minutes ahead. To overtake it the minute-hand must move over $1\frac{1}{11}$ of 10 minutes, or $10\frac{10}{11}$ minutes. The hands are together at $10\frac{10}{11}$ minutes past 2 o'clock.

13. At what time between 3 and 4 o'clock is the minute-hand just $2\frac{1}{2}$ minutes ahead of the hour-hand?

Analysis.—At 3 o'clock the minute-hand is behind the hour-hand 15 minutes. To be $2\frac{1}{2}$ minutes ahead it must gain $17\frac{1}{2}$ minutes on the hour-hand; hence, the minute-hand must move over $1\frac{1}{11}$ of $17\frac{1}{2} = 19\frac{1}{11}$ minutes. At $19\frac{1}{11}$ minutes past 3 o'clock the minute-hand will be just $2\frac{1}{2}$ minutes ahead of the hour-hand.

14. At what time between 4 and 5 o'clock do the hands of a watch make with each other an angle of 45° ?

Analysis.—At 4 o'clock the minute-hand is 20 minutes behind the hour-hand. To make with it an angle of 45° the minute-hand must be either $7\frac{1}{2}$ minutes behind, or $7\frac{1}{2}$ minutes ahead of the hour-hand. In the first case the minute-hand must gain $20 - 7\frac{1}{2} = 12\frac{1}{2}$ minutes, and must pass over $1\frac{1}{11}$ times $12\frac{1}{2} = 13\frac{7}{11}$ minutes.

In the second case the minute-hand must gain $20 + 7\frac{1}{2} = 27\frac{1}{2}$ minutes, and must pass over $1\frac{1}{11}$ of $27\frac{1}{2} = 30$ minutes. At $13\frac{7}{11}$ minutes past 4 o'clock, or at 30 minutes past 4 o'clock, the hands will make an angle of 45° with each other.

15. At what time between 5 and 6 o'clock is the

minute-hand as many minutes in advance of VI. as the hour-hand is behind VI.?

Analysis.—At 5 o'clock the minute-hand is at XII., and the hour-hand at V. Regarding the distance the hour-hand moves beyond V. as *one space*, the distance the minute-hand moves beyond XII. is *twelve* of these equal spaces. But, as the minute-hand lacks one of these equal spaces of being at VII., the distance between XII. and VII., or 35 minutes, must be thirteen of the spaces the hour-hand has moved; or the hour-hand has moved $\frac{1}{13}$ of $35 = 2\frac{2}{13}$ minutes. Then, the minute-hand has moved 35 minutes— $2\frac{2}{13}$ minutes, or $32\frac{4}{13}$ minutes. Then, $32\frac{4}{13}$ minutes past 5 o'clock is the time required.

16. A man looked at his watch and found the time to be between 5 and 6 o'clock. Within an hour he looked again, and found the hands had exchanged places. What was the exact time when he first looked at his watch?

Analysis.—There are three possible positions of the two hands with reference to each other, but only one of them will satisfy the second condition of the problem. The minute-hand was *ahead* of the hour-hand when he first looked at his watch. Regarding the distance the hour-hand moved from its first position to the first position of the minute-hand, as *one space*, the minute-hand moved over twelve of these spaces in passing from its first position to its second position. As it then lacked *one space* of being at its first position again, the whole dial, or 60 minutes, equals thirteen times the distance the hour-hand has passed over. Hence, the hour-hand passed over $\frac{1}{13}$ of 60 minutes, or $4\frac{8}{13}$ minutes. This is the number of minutes the minute-hand was *ahead* of the hour-hand when he first looked at his watch.

At 5 o'clock the minute-hand was 25 minutes behind the hour-hand, and to be ahead of it $4\frac{8}{13}$ minutes, the minute-hand must gain 25 minutes $+ 4\frac{8}{13}$ minutes, or $29\frac{8}{13}$ minutes, and must pass over $\frac{11}{13}$ of $29\frac{8}{13} = 32\frac{4}{13}$ minutes.

The time he looked at his watch was $32\frac{4}{13}$ minutes past 5 o'clock

17. What time of day is it when the time past noon is $\frac{1}{4}$ of the time from now till midnight?

Analysis. — Since the time past noon is $\frac{1}{4}$ of the time from *now* till midnight, the whole time from noon till midnight, — that is 12 hours, — must be $\frac{4}{3}$ of the time from *now* till midnight; hence, the time past noon, or the time required, is $\frac{1}{3}$ of 12 hours, or 2 hours and 24 minutes. The time required is 24 minutes past 2 o'clock.

18. A man agreed to work 20 days at \$5 a day, and to forfeit \$2 every day he was idle. At the end of the time he received \$72; how many days did he work?

Analysis. — Had he worked the whole time he would have received \$100; but as he received only \$72, he lost by his idleness \$28. He lost each day $\$5 + \$2 = \$7$. To lose \$28, he must have been idle 4 days. Then he worked $20 - 4 = 16$ days.

19. A fox is 50 leaps ahead of a hound, and takes 4 leaps in the time the hound takes 3; but 2 of the hound's leaps equal 3 of the fox's leaps. How many leaps must the hound take to catch the fox?

Analysis. — Since 2 leaps of the hound equal 3 leaps of the fox, 3 leaps of the hound equal $4\frac{1}{2}$ leaps of the fox. But these 3 leaps of the hound are taken in the same time the fox takes 4; hence the hound gains $\frac{1}{2}$ a leap in every 3 leaps, or he must take 6 leaps to gain one whole leap on the fox. Then, to gain 50 leaps the hound must take 300 leaps.

20. A man dying in Boston left an estate valued at \$91000 to be divided between his widow, son, and daughter. The terms of his will provided that in case his daughter, who was in Paris, should return, and not the son, the widow should take $\frac{2}{3}$ of the estate and the daughter the remainder; but in case the son, who was in London, should return, and not the daughter, the

widow should take $\frac{1}{3}$ of the estate and the son the remainder. Both son and daughter returned. How much, in equity, should each receive?

Analysis. — Since by the terms of the will the son was to get *twice* as much as the widow, and the widow *twice* as much as the daughter, the son was to get *four* times as much as the daughter; hence they were all to get *seven* times as much as the daughter. The daughter should get $\frac{1}{7}$ of \$91000, or \$13000, the widow \$26000, and the son \$52000.

21. Another estate was to be divided on the above terms; but as the son and daughter both returned, the widow received \$2400 less than would have been her portion had the daughter alone returned. Find the value of the whole estate.

Analysis. — By the first condition the widow was to get $\frac{2}{3}$ of the estate; by the last condition, she actually received $\frac{1}{3}$ of the estate. She received $\frac{2}{3} - \frac{1}{3}$, or $\frac{1}{3}$, less by the last than by the first condition. The estate must have been $\frac{2}{1}$ of \$2400, or \$6300.

22. A, B, and C have \$16800 invested in business together. Their interests are in the ratio of $\frac{2}{7}$, $\frac{3}{8}$, and $\frac{5}{6}$, respectively. B dies, and the remaining partners pay his widow \$2700 cash for his interest in the business. What is each one's share of the remaining capital?

Analysis. — After paying for B's interest, the capital was \$16800—\$2700=\$14100. The interests of A and C are now in the ratio of $\frac{2}{7}$ and $\frac{3}{6}$; hence A owns $\frac{1}{2}$ of \$14100=\$3600, and C owns $\frac{1}{2}$ of \$14100=\$10500.

23. A merchant bought goods at 20 cents a yard, which he sold at a net profit of 40 %, after allowing his customers a discount of $12\frac{1}{2}$ %. What were the goods marked?

Analysis. — 40 % of .20 is .08. His selling price was $.20 + .08 = .28$. $12\frac{1}{2}$ % is $\frac{1}{8}$; hence his selling price was $\frac{1}{8}$ of his asking price; or the asking price was $\frac{8}{7}$ of .28, or .32.

24. When gold was quoted at 130, goods sold at .52. What were they worth in gold?

Analysis. — Since \$1.30 in currency is worth \$1.00 in gold, 1 cent in currency is worth $\frac{100}{130} = \frac{10}{13}$ of a cent in gold, and 52 cents in currency are worth 52 times $\frac{10}{13} = 40$ cents in gold.

25. When gold is quoted at 210, goods are purchased in England at 30 cents a yard. What must they be marked to make a profit of $11\frac{1}{5}$ %?

Analysis. — Since the goods are worth 30 cents in gold, they are worth $\frac{210}{100}$, or $\frac{21}{10}$, of 30 cents, or 63 cents in currency. To gain $11\frac{1}{5}$ % they must be sold for $\frac{70}{100}$ of 63 = 70 cents a yard.

26. When 6 % bonds are bought at 90, what is the rate of income?

Analysis. — If what cost \$90 yields \$6, the rate of income is $\frac{6}{90} = \frac{1}{15}$ of 100 %, or $6\frac{2}{3}$ %.

27. What is the market price of U. S. 5 % bonds that yield 4 % on the investment?

Analysis. — Since the rate of income is less than the rate of the bond, the bonds are at a premium, and must be quoted at $\frac{4}{5}$ of their par value, or at 125.

28. Which is the better investment, U. S. 6's at 95, or 7-30's at 115? How much?

Analysis. — U. S. 6's bought at 95 yield 6.3158 %. U. S. 7-30's bought at 115 yield 6.3478 %. Hence, U. S. 7-30's at 115 are a better investment than U. S. 6's at 95, by .032 %.

29. A square field contains as many acres as there are rods around it. How many acres does it contain?

Analysis.—The number of acres in the field equals four times the number of rods on one side of it. Also, the number of acres in the field equals $\frac{1}{160}$ of the square of the number of rods on one side of it. Hence, 4 times 160 rods gives the number of rods on one side of the field. Then, the field must have contained $(640)^2 \div 160 = 2560$ acres.

30. Four men did a piece of work for a certain price. In dividing the money A took \$60.06, and $\frac{1}{10}$ of the remainder; B took \$70.07, and $\frac{1}{10}$ of the remainder; C took \$80.08, and $\frac{1}{10}$ of the remainder; D took what was left. By this division they received equal amounts. What was the whole amount received?

Analysis.—D's portion was $\frac{9}{10}$ of the last remainder, and C's portion was $\frac{1}{10}$ of the same remainder, plus \$80.08. Hence, \$80.08 was $\frac{1}{10}$ of this remainder. Then, $\frac{1}{10}$ of this remainder was $\frac{1}{9}$ of \$80.08, or \$10.01; and $\frac{9}{10}$ of this remainder, or D's share, was 9 times \$10.01, or \$90.09. The whole amount was \$360.86.

31. A owed B \$900, due in 4 months. B offered to allow a discount of 4 % for ready money, and A immediately paid \$696 in cash. How much was still due?

Analysis.—The amount paid was $\frac{96}{100} = \frac{24}{25}$ of the amount liquidated; hence the amount liquidated was $\frac{25}{24}$ of \$696, or \$725. The balance still due was \$900 — \$725 = \$175.

32. A man bought cloth $1\frac{3}{4}$ yards wide at \$5.70 per yard; the cloth, getting wet, shrunk 5 % in length and $2\frac{1}{2}$ % in width. At what price per square yard must he sell it to gain 19 %.

Analysis.—He bought $\frac{7}{4}$ yards. Allowing for shrinkage 5 %, or $\frac{1}{20}$ in length, there remained $\frac{13}{8}$ of $\frac{7}{4}$ yards. Allowing for shrinkage $2\frac{1}{2}$ %, or $\frac{5}{40}$ in width, there remained $\frac{3}{4}$ of $\frac{13}{8}$ of $\frac{7}{4}$ yards. Since this number of yards cost \$5.70, one yard cost $\frac{3}{4} \times \frac{13}{8} \times \frac{7}{4} \times \$5.70$. Then, to gain 19 %, one yard must sell for $\frac{119}{100} \times \frac{3}{4} \times \frac{13}{8} \times \frac{7}{4} \times \$5.70$. Cancelling common factors, this reduces to \$4.18 $\frac{1}{3}$.

33. A man bought goods at 13 cents a yard on 8 months' credit, and sold them the same day for 12 cents cash. What was his rate of gain, money being worth 6 %?

Analysis.—The cash value of goods bought at 13 cents, on 8 months' credit, money being worth 6 % per annum, is $.13 \div 1.04 = .125$. Then, the loss on each yard was $.125 - .12 = .005$, or $\frac{1}{200}$ of the cost; that is, 4 %.

34. Sold a horse so that $\frac{4}{5}$ of the gain equaled $\frac{2}{15}$ of the cost. What was the gain per cent?

Analysis.—Since $\frac{4}{5}$ of the gain equaled $\frac{2}{15}$ of the cost, the whole gain equaled $\frac{1}{5}$ of the cost, or $\frac{1}{5}$ of 100 % = $16\frac{2}{3}$ %.

35. What must goods be marked, so that, allowing 8 % for expense in handling, 4 % for bad debts, and an average credit of 3 months, the net gain may be 15 %, money being worth 6 % per annum?

Analysis.—The original cost of \$1 worth of goods plus the cost of handling gives \$1.08. 15 % of 1.08 = .162. The selling price without reference to bad debts and the term of credit, must be $\$1.08 + .162 = \$1.242$. Allowing for bad debts, the asking price must be $\frac{2}{3}$ of $\$1.242 = \$1.29375$. Allowing for term of credit the selling price must be $\$1.29375 \div .825 = \$1.55\frac{1}{3}$. And $\$1.55\frac{1}{3} - \$1 = 55\frac{1}{3}$ % advance.

36. Three men carry a timber 30 feet in length, of uniform size and density; two of the men lift with a handspike, and one at the end; how far from the other end must the handspike be placed so that the men shall lift an equal weight?

Analysis.—If the entire weight of the timber be regarded as massed at the center, the two men must be $\frac{1}{2}$ the distance from the weight that the one man is; that is, $\frac{1}{2}$ of 15 feet; hence, they must place the handspike $7\frac{1}{2}$ feet from the other end of the timber.

37. Five men carry a timber 30 feet in length, of uniform size and density; four of the men lift with a handspike, and one at the end; how far from the other end must the handspike be placed so that the several men shall lift an equal weight?

Analysis. — If the entire weight be regarded as massed at the center, the four men must be $\frac{1}{4}$ the distance from the weight that the one man is, that is $\frac{1}{4}$ of 15 feet; hence the handspike must be placed $3\frac{1}{4}$ feet from the center, or 11 feet 3 inches from the other end.

38. A man has just 2 spare hours. How many miles may he ride in a coach that travels 12 miles an hour, so that he can return, walking back at the rate of 4 miles an hour?

Analysis. — He rides 1 mile in $\frac{1}{12}$ of an hour, and walks one mile in $\frac{1}{4}$ of an hour. Then he can ride 1 mile and return in $\frac{1}{12} + \frac{1}{4} = \frac{1}{3}$ or $\frac{1}{3}$ of an hour. In 2 hours he can ride as many miles and return as $\frac{1}{3}$ is contained in 2, or 6 miles.

39. If 65 pounds of sea-water contain 2 pounds of salt, how much fresh water must be added so that 25 pounds of the sea-water shall contain $\frac{1}{4}$ of a pound of salt?

Analysis. — By the last condition 25 pounds of sea-water contained $\frac{1}{4}$ of a pound of salt, hence 100 pounds of sea-water are required to contain 1 pound of salt, and to contain 2 pounds of salt 200 pounds of sea-water are required. Since at first 65 pounds of sea-water contain 2 pounds of salt, there have been added $200 - 65 = 135$ pounds of fresh water.

40. A man plows 1 acre of land for every 3 horses he keeps, and has 1 acre of pasture land for every 2 horses. How many horses can he keep on 25 acres?

Analysis. — He kept $\frac{1}{3}$ of an acre of plowed land and $\frac{1}{2}$ of an

acre of pasture land for every horse he had. Then, to keep 1 horse he must have $\frac{1}{2} + \frac{1}{2} = \frac{1}{1}$ of an acre of land. Hence, on 25 acres he could keep $25 \div \frac{1}{1}$, or 30 horses.

41. In a mass of copper and tin containing 80 pounds, the tin is to the copper as 3 to 7. How much copper must be added to the mass, so that the tin shall be to the copper as 4 to 11?

Analysis.—In the first instance the tin is $\frac{3}{10}$ of the mass. The tin in the second instance is 24 pounds, which is $\frac{4}{15}$ of the mass; hence the new mixture contained 90 pounds. Then 10 pounds must have been added.

42. A man bought eggs at 18 cents a dozen; had he bought 5 more for the same money, they would have cost him $2\frac{1}{2}$ cents less a dozen. How many eggs did he buy?

Analysis.—Had he bought 5 more eggs, the lot would have cost $15\frac{1}{2}$ cents a dozen. Now, as the price per dozen diminishes as the number of eggs purchased increases, 18 cents is the same part of $15\frac{1}{2}$ cents that the number of eggs bought in the last case is of the number of eggs bought in the first case; that is, it is $\frac{3}{11}$ of the number bought in the first case. Hence, 5 eggs must be $\frac{5}{11}$ of the original purchase, and 31 eggs the number bought at first.

43. A father left a number of children, and a certain sum to be divided among them. The first was to receive \$100 and $\frac{1}{10}$ of the remainder; the second, \$200 and $\frac{1}{10}$ of the remainder; the third, \$300 and $\frac{1}{10}$ of the remainder, and so on. By this division each received the same sum. How many children were there, and what amount was divided among them?

Analysis.—The tenths of any two consecutive remainders differ by \$100, since any two consecutive legacies differ by \$100; hence the whole of any two consecutive remainders differs by \$1000.

The first remainder, then, differed by \$800 from the amount left, before the second legacy of \$200 was paid. Then \$800 was $\frac{1}{10}$ of the first remainder, and the whole of that remainder was \$8000; hence the whole fortune was \$8100. It will be readily seen that each child received \$900 dollars; hence, there were 9 children.

44. A and B had together a certain number of guineas. The sum of the squares of the number each had equals 208, and the difference of these is 80. If A had the greater number, how many had each?

Analysis. — Had they had an equal number of guineas, the sum of their squares must have been 288 or 128; hence the square of A's number must be 144, and of B's number 64. A, then, had 12 guineas, and B had 8 guineas.

45. If 12 oxen eat $3\frac{1}{8}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen are required to eat 24 acres of grass in 18 weeks, original quantity and growth?

ANALYTICAL STEPS. — 1. Find the number of oxen required to consume the growth on 10 acres in 9 weeks.

2. Find the number of oxen required to consume the original quantity on 10 acres in 9 weeks.

3. Find the number of oxen required to consume the growth on 24 acres.

4. Find the number of oxen required to consume the original quantity on 24 acres in 18 weeks.

Analysis. — Since $3\frac{1}{8}$ acres pasture 12 oxen 4 weeks, 10 acres pasture 36 oxen 4 weeks; but these 10 acres pasture only 16 oxen for 9 weeks. Now, 10 acres pasture 21 oxen 9 weeks; hence 5 oxen consume the growth on 10 acres in 5 weeks, and 9 oxen consume the 9 weeks' growth on 10 acres. Then 12 oxen eat the original quantity on 10 acres in 9 weeks. By the third condition, then, the original quantity on 24 acres will keep $14\frac{2}{3}$ oxen 18 weeks, and the growth on 24 acres during the 18 weeks will keep $21\frac{2}{3}$ oxen. Therefore, 24 acres will pasture 36 oxen 18 weeks.

46. If 12 oxen eat $3\frac{1}{2}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, in how many weeks can 36 oxen eat 24 acres, the grass growing uniformly during the grazing?

ANALYTICAL STEPS. — 1. Find the number of oxen that eat the growth on 24 acres.

2. Find the number of oxen that eat, in the required time, the original quantity on 24 acres.

3. Find the time in which one ox can eat the original quantity on 24 acres.

4. Find the *time* it will take the number of oxen required to eat the original quantity on 24 acres.

Analysis. — Since 12 oxen can eat the grass on $3\frac{1}{2}$ acres in 4 weeks, 36 oxen can in the same time eat the grass on 10 acres; hence 16 oxen can eat the grass on 10 acres in 9 weeks. Then 9 oxen can eat the growth on 10 acres in 9 weeks, and 12 oxen can eat the original quantity on 10 acres in 9 weeks. Since 9 oxen can eat the growth on 10 acres, $21\frac{1}{2}$ oxen can eat the growth on 24 acres; hence $14\frac{1}{2}$ oxen can, in the required time, eat the original quantity on 24 acres. Now, 12 oxen can, in 9 weeks, eat the original quantity on 10 acres, and 1 ox can eat this quantity in 108 weeks. Then 1 ox can eat the original quantity on 24 acres in $259\frac{1}{2}$ weeks, and $14\frac{1}{2}$ oxen can eat this quantity in $259\frac{1}{2} \div 14\frac{1}{2} = 18$ weeks. Therefore, 24 acres will pasture 36 oxen 18 weeks.

47. If 10 acres pasture 21 oxen 9 weeks, and 12 oxen eat the grass on $3\frac{1}{2}$ acres in 4 weeks, how many acres will pasture 36 oxen for 18 weeks?

ANALYTICAL STEPS. — 1. Find the number of oxen required to eat 9 weeks' growth on 10 acres.

2. Find the number of oxen required to eat the original quantity on 10 acres in 9 weeks.

3. Find the number of acres of original quantity required to keep 36 oxen 18 weeks.

4. Find the number of acres of growth required to keep 36 oxen.

5. Find the number of oxen these acres of original quantity will pasture for 18 weeks.

6. Find the number of oxen these acres of growth will pasture 18 weeks.

7. Find the acres of original quantity 1 ox consumes in 18 weeks.

8. Find the acres of original quantity consumed in 18 weeks, by the number of oxen found in step 5.

Analysis. — To eat 9 weeks' growth on 10 acres requires 9 oxen. Hence 12 oxen are required to eat, in 9 weeks, the original quantity on 10 acres. Then, the original quantity on 60 acres, or the growth on 40 acres, will pasture 36 oxen for 18 weeks. Since it takes $\frac{2}{3}$ as many acres of original quantity as of growth to pasture 36 oxen, it must require only $\frac{2}{3}$ as many oxen to consume the original quantity as to consume the growth on the same number of acres. Then, to consume the original quantity and the growth must require $\frac{5}{3}$ the number of oxen that consume the growth. Hence, $\frac{2}{3}$ of 36, which is 24 oxen, eat the growth, and 12 oxen eat the original quantity on the acres required. The original quantity on 10 acres keeps 12 oxen 9 weeks; hence the original quantity on $1\frac{1}{2}$ acres keeps one ox 18 weeks, and on $24\frac{1}{2}$ acres this quantity will keep 14 oxen 18 weeks.

48. Four men bought a grindstone 60 inches in diameter. How much must each grind off to get $\frac{1}{4}$ of the stone, if no allowance is made for the eye?

Analysis. — As the stone is of uniform thickness, we need regard only the surface of the portion ground off. Since circles are to each other as the squares of their diameters, or radii, we proceed as follows:

a. After the first man his ground off his share, $\frac{1}{4}$ of the stone remains, and its diameter is

$$60\sqrt{\frac{3}{4}} \text{ or } 30\sqrt{3}, \text{ which is } 51.96+ \text{ inches.}$$

b. After the second has ground off his share, $\frac{2}{4}$ of the stone remains, and its diameter is

$$60\sqrt{\frac{2}{4}} \text{ or } 30\sqrt{2}, \text{ which is } 42.426+ \text{ inches.}$$

c. After the third man has ground off his share, $\frac{3}{4}$ of the stone remains, and its diameter is

$$60\sqrt{\frac{1}{4}} \text{ or } 30\sqrt{1}, \text{ which is } 30 \text{ inches.}$$

Then, the first man ground off.....	60.00 — 51.96 =	8.04 inches.
The second man ground off.....	51.96 — 42.426 =	9.534 inches.
The third man ground off.....	42.426 — 30. =	12.426 inches.
The fourth man ground off.....		30. inches.
		60 inches.

49. Four men bought a grindstone 40 inches in diameter. How much of the diameter must each man grind off in order that they may share the stone equally, 4 inches of the diameter being allowed for the eye?

Analysis. — The area of the eye in this case is one-hundredth of the area of the stone, hence —

a. After the first man has ground off his share there remains $\frac{3}{4}$ of $\frac{39}{100}$, or $\frac{117}{100}$ of the stone. Restoring the value of the eye, to preserve the similarity of the remainder to the whole stone, we have the ratio 301 to 400. Then the diameter of this remainder is

$40\sqrt{\frac{117}{100}}$, or $2\sqrt{301}$, which is 34.6986 inches.

b. After the second has ground off his share, there remains $\frac{1}{2}$ of $\frac{39}{100}$, or $\frac{195}{100}$ of the stone. Restoring the value of the eye, we find the ratio of this remainder to the whole stone to be 203 to 400. Then, the diameter of this remainder is

$40\sqrt{\frac{195}{100}}$ or $2\sqrt{202}$, which is 28.4252 inches.

c. After the third man has ground off his share there remains $\frac{1}{4}$ of $\frac{39}{100}$, or $\frac{99}{100}$ of the stone. Again restoring the value of the eye, we find the ratio of this remainder to the whole stone to be 103 to 400. The diameter of this remainder is

$40\sqrt{\frac{103}{100}}$ or $2\sqrt{103}$, which is 20.2976 inches.

The first man ground off.....	40 — 34.6986 =	5.3014 inches.
The second man ground off.....	34.6986 — 28.4252 =	6.2734 inches.
The third man ground off.....	28.4252 — 20.2976 =	8.1276 inches.
The fourth man ground off..	20.2976 — 4 (the eye) =	16.2976 inches.
		36 inches.

50. Three women own a ball of thread 3 inches in diameter; how much must each wind off to get $\frac{1}{3}$ of the thread?

Analysis.—After the first one winds off her share $\frac{3}{4}$ of the thread remains, and the diameter of the ball is now $3\sqrt[3]{\frac{2}{3}}$, or $\sqrt[3]{18}$. The $\sqrt[3]{18}$ is 2.620697.

b. After the second one winds off her share $\frac{1}{3}$ of the thread remains, and the diameter of the ball is $3\sqrt[3]{\frac{1}{3}}$, or $\sqrt[3]{9}$. The $\sqrt[3]{9}$ is 2.080083. Hence,

The first woman winds off..... $3 - 2.620697 = .379303$ inches.

The second woman winds off $2.620697 - 2.080083 = .540604$ inches.

The third woman winds off $3 - .919907 = 2.080093$ inches.

3 inches.

51. Three women bought a ball of silk 10 inches in diameter, which was wound on a ball of wood 1 inch in diameter. How much of the diameter must each wind off in order to share the silk equally?

Analysis.—The ball in the center was $\frac{1}{1000}$ of the whole ball; hence,

a. After the first woman winds off her share of the thread, $\frac{3}{4}$ of $\frac{999}{1000}$, or $\frac{666.75}{1000}$ of the thread remains. Restoring the value of the ball of wood, to preserve the similarity of the solids, we find the ratio of this remainder to the whole ball to be 667 to 1000. Then the diameter of this remainder is

$10\sqrt[3]{\frac{667}{1000}}$, or $\sqrt[3]{667}$, which is 8.737 inches. Hence,

The first woman winds off..... $10 - 8.737 = 1.263$ inches.

The second woman winds off..... 1.799 inches.

The third woman winds off..... 5.938 inches.

9 inches.

52. A father left 4 sons, whose ages at the time of his death were 9, 13, 15, and 17 years. His estate, which was valued at \$6425, was to be divided among them so that their shares, at 5 % simple interest, should amount to equal sums when the children became 21 years of age. Find the share of each.

Analysis.—These shares, placed at 5 % simple interest,

amount to equal sums in 12, 8, 6, and 4 years respectively; hence the estate must be divided in the ratio of the present worth of \$1 for these several periods at the given rate per cent.

The present worth of \$1, at 5 % for 12 years, is \$.62 $\frac{1}{2}$.

The present worth of \$1, at 5 % for 8 years, is \$.71 $\frac{1}{2}$.

The present worth of \$1, at 5 % for 6 years, is \$.76 $\frac{1}{2}$.

The present worth of \$1, at 5 % for 4 years, is \$.83 $\frac{1}{2}$.

These present worths are in the ratio of the following whole numbers: 34125, 39000, 42000, and 45500. Hence,

The share of the youngest child was \$1365.

The share of the second child was \$1560.

The share of the third child was \$1680.

The share of the oldest child was \$1820.

\$6425.

53. A servant was to receive \$70 and a suit of clothes for a year's service; but he left his employer at the end of 9 months, receiving the suit of clothes and \$42 in cash. What was the value of the suit of clothes?

Analysis. — He was to receive the clothes and \$70 for 12 months' service, but he received the clothes and \$42 for 9 months' service; he therefore lost \$28 by not working the remaining 3 months. Then his services were worth \$9 $\frac{1}{3}$ a month in cash. For 9 months his services were worth \$84 in cash. As he received only \$42, the clothes were worth \$84 — \$42 = \$42.

MISCELLANEOUS PROBLEMS.

1. In what time will any principal double itself at 5 % simple interest?
2. In what time will any principal double itself at 6 % simple interest?
3. In what time will any principal become four times itself at 6 % simple interest?

4. In what time will any principal become one thousand times itself at 100 % simple interest?

5. At what rate will any principal double itself in 25 years?

6. In what time will any principal become $2\frac{1}{2}$ times itself at $2\frac{1}{2}$ % simple interest?

7. A man commenced business with \$5800, and gained during the year \$870. Find the rate of gain.

8. An agent sold cotton at $2\frac{1}{2}$ % commission, and invested $\frac{3}{4}$ of the amount in sugar at $2\frac{1}{3}$ % commission; the balance, which was \$1915.225, was remitted. What was the commission, the value of the cotton, and the cost of the sugar?

9. A owes B \$480, due in 12 months, and B owes A \$720, due in one year and six months. If A should pay his indebtedness at once, when should B pay his?

10. On goods sold at \$5 per yard there is a gain of 25 %. What would be the gain % on the same goods when sold at \$7 per yard?

11. When money was worth 6 %, I bought a watch for \$30 cash, and sold it for \$35 on 8 months' credit. What was my gain?

12. A sells B tea worth 45 cents cash, at 48 cents. What should B charge A for coffee worth 9 cents?

13. A man bought cloth for 8 % less than its value, and sold it for 10 % more than its value, thereby gaining $88\frac{1}{3}$ cents. What did it cost him?

14. A broker bought stocks for me at a discount of 30 %, and sold them at 10 % discount. His brokerage

on the purchase was $1\frac{1}{2}\%$, and on the sale $1\frac{1}{3}\%$. I gained on the whole transaction \$754.37 $\frac{1}{2}$; each share being \$25; how many shares were bought?

15. Property valued at \$1980, is insured at 1%, so as to cover the valuation and the premium. Find the amount of the policy?

16. The *ad valorem* duty on 2000 yards of broad-cloth at 25 % is \$2000. At what price per yard are the goods invoiced?

17. A merchant purchased cloth to the amount of \$1250, and silk goods to the amount of \$900; he sold the cloth at a profit of 20 %, and the silk at a loss of 10 %. Find his rate of gain?

18. At what rate must I buy a 6 % stock to make my investment as good as a 5 % stock bought at a discount of 25 %?

19. What is the cost, including brokerage, of \$9000 U. S. 6's at 112 $\frac{1}{4}$?

20. An 8 % stock sells at 112; what rate of interest does it yield the purchaser?

21. I wish to purchase a 5 % stock, so as to make the investment yield me 7 %; how much can I pay for the stock, including brokerage at $\frac{1}{2}\%$?

22. A's money is 50 % more than B's; B's money is what % less than A's?

23. When gold is at 110, what are greenbacks worth?

24. A piece of property can be purchased for \$5000 in gold when the premium on gold is 12 %. What is the value of the same in currency?

25. If I buy U. S. 6's at 112 $\frac{1}{2}$, including brokerage

at $\frac{3}{4}\%$, and sell the gold interest at $107\frac{3}{4}$ currency, what % will I make on the investment?

26. A broker sells 30 shares of bank stock at $96\frac{1}{2}$, and 120 shares of railroad stock at 105, retaining the brokerage at $\frac{3}{4}\%$; how much does he pay to his principal?

27. What quantity of water must be added to 63 gallons of wine, worth \$27, so that it can be sold for 40 cents a gallon, at a profit of 10 cents a gallon?

28. A and B can do a piece of work in 20 days; A and C can do it in 15 days; B and C can do it in 12 days. In what time can they all do it together?

29. A and B can do $\frac{3}{8}$ of a piece of work in one day; A and C can do $\frac{1}{8}$ of it in one day; B and C can do $\frac{5}{12}$ of it in one day. In what time can they all do it?

30. A, B, C, and D engage to do a piece of work; A, B, and C can do it in 18 days; B, C, and D can do it in 20 days; A, C, and D can do it in 24 days; A, B, and D can do it in 27 days. In what time can they do it jointly, and separately?

31. If 20 % of what I receive for goods is gain, what is my gain %?

32. John had 36 marbles; James had 25 % more than John; after playing, James had one more than 50 % more than John. How many marbles did James win?

33. John said to Charles, "You have 8 marbles, and if $\frac{1}{18}$ of what we both have be added to what you have, the sum will be equal to what I have." How many marbles had John?

34. If $\frac{2}{7}$ of the selling price is gain, what is the gain %?

35. Bought 126 gallons of molasses for \$35, a part of it having been wasted, and the remainder sold at $32\frac{1}{2}$ cents a gallon, my loss was 7 %. How many gallons were wasted?

36. How much water must be mixed with 100 gallons of alcohol, worth 7s. 6d. per gallon to reduce its value to 6s. 3d. per gallon?

37. By selling a piece of damaged cloth at \$2.50 per yard, I sustain a loss of $16\frac{2}{3}$ %. What was my loss on 20 yards?

38. An agent sold a lot of cloth for \$200, at a profit of 20 % to the owner. If he had sold it for \$175, the commission in each case being 2 %, what would have been the owner's net profit %?

39. A merchant paid \$2000 for flour. He sells on a credit of 6 months, at 7 % profit. If money is worth 6 %, what was his selling price?

40. How many gallons of wine, at \$4.50 and at \$5 per gallon, must be mixed with 6 gallons at \$4, and 6 gallons at \$3 per gallon, to make a mixture worth $\$4.33\frac{1}{3}$ per gallon?

41. A public square is surrounded by a walk 2 rods in width, containing one acre. Find the side of the square?

42. An agent sold property for \$345.30, after taking out his commission at $2\frac{1}{2}$ %. Having kept the money for 8 months and 21 days, he pays it over to his employer. Money being worth 8 %, find the amount paid over.

43. One year from date, I promise to pay Joseph Mason \$500, with interest from date, for value received.

N. Y., May 1st, 1860.

A. MUNSON.

Indorsements on above note : May 1st, 1861, \$200 ; June 10th, 1862, \$100.50 ; July 15th, 1863, \$50.75. What was due September 20th, 1864 ?

44. A man borrowed \$5000, which he immediately invested in real estate. In six months he sold the property, on a credit of 12 months, with interest, for \$7500. Money being worth 6 %, what was his profit ?

45. If a body move $\frac{1}{8}$ of a foot the first second, and $\frac{1}{8}$ of this distance the second second, and so on each succeeding second of time, through what entire distance would it move, provided it never stopped ?

46. A man sold a quantity of wheat at \$1.08 per bushel, at a profit of 20 % ; he afterwards sold a lot of the same for \$31.05, at a profit of 50 %. How many bushels in the last lot ?

47. I sold pens at 11 shillings per thousand, at a profit of $\frac{2}{3}$ of their cost. I afterwards sold them at 13s. 6d. per thousand ; what % did I clear at this price ?

48. A, B, and C jointly inherit under the terms of a will, the sum of \$100000. Their shares are in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively ; but, C having died, the legacy was divided between A and B. How much did each receive ?

49. The commission, at 5 %, on a consignment of sugar was \$95. The consignor made a profit of 25 %. What was the amount of the consignment ?

50. When cloth is sold at 14 cents per yard, there is a gain of 75 cents on each piece ; but when it is sold at $12\frac{1}{2}$ cents per yard, there is a loss of 15 cents on each piece. How many yards in each piece ?

51. If $16\frac{1}{3}\%$ of what I receive is gain, what is my gain %?

52. A man sells goods at a gain of 30 %, and takes a note for the amount of the sale. If he sells the note at 15 % less than its face value, what is his gain %?

53. A lot of sheep and lambs were bought for \$1000. Each sheep cost \$3, and each lamb \$2. $\frac{1}{4}$ of the sheep, and $\frac{1}{2}$ of the lambs were sold, at a profit of \$37.50, for \$387.50. How many of each were bought?

54. A drover, having calves, sheep, and hogs, was met by an inquisitive fellow, who asked the number of animals in his drove. The drover replied: " $\frac{2}{5}$ of my drove are sheep, $\frac{3}{8}$ of the remainder are hogs, and the rest calves; if I sell my sheep for \$2.50 per head, my hogs at \$3.50 per head, and my calves at \$5 per head, I shall sell the drove for \$519, at a profit of \$119." How many of each kind were in the drove?

55. A servant received for his yearly wages \$40 and a suit of clothes. At the end of 5 months he received the suit of clothes and \$6 $\frac{1}{4}$ in money. Find the value of the suit?

56. A farmer allows one acre of pasture for every 5 sheep, and one acre of plowed land for every 3 sheep. How many sheep can be kept on 325 acres?

57. A grocer has two kinds of sugar. Two pounds, one of each kind, are worth 20 cents; but if 3 pounds of the first and 5 pounds of the second be taken, a pound of the mixture will be worth 11 cents. Find the value of a pound of each kind.

58. A farmer rents a farm for \$245; the tillable land

being valued at \$2 an acre, and the pasture land at \$1.40 an acre. The number of acres of tillable land is to the excess of the tillable land above the pasture land as 14 to 9. How many acres of each kind were there?

59. A and B kill a deer, whose weight they determine in the following manner: A, whose weight is 147 pounds, just balances the deer suspended at the other end of a pole which rests upon the limb of a tree. Without changing the *point* of support, B, whose weight is 192 pounds, just balances the deer suspended at A's end of the pole. What is the weight of the deer?

60. A man bought eggs at 20 cents a dozen; had he bought 6 more eggs for the same money, they would have cost him $1\frac{1}{8}$ cents a dozen less. How many eggs did he buy?

61. Three painters do a piece of work, no two of them working at the same time. The first receives \$6 per day, the second receives \$4 per day, and the third receives \$3 per day. They complete the work in 100 days, and receive equal sums of money. How many days did each man work?

62. A merchant bought 60 barrels of flour, and sold 20 of them so as to gain a certain rate per cent. He then sold 15 barrels so as to gain twice as much per cent., and the remaining 25 barrels so as to gain three times his first rate per cent. He thus cleared 20 per cent. on the whole lot. Find the gain per cent. on each sale.

63. A ship, containing 150 hogsheads of wine, pays for toll at the Sound, the value of 2 hogsheads of wine,

lacking \$6; and another ship, containing 240 hogsheads of wine, pays, at the same rate, the value of 2 hogsheads of wine and \$18 more. Find the value of a hogshead of wine.

64. An agent, on assuming the liquor agency of a town, received \$58 in cash, and liquors valued at \$59.50. During the year he paid \$282.85 for stock, and received on sales \$293.55. At the close of the year he delivered liquors valued at \$81 to the town. He was to receive \$69 for his services. Did the agent owe the town, or did the town owe the agent, and how much? Did the town make or lose, and how much?

65. A prisoner escaped from prison at 6 A. M., and at 4 P. M. the sheriff started in pursuit, gaining 3 miles per hour. At midnight the sheriff met an express traveling at his own rate, who reported that he met the prisoner 24 minutes before 10 o'clock. At what time did the sheriff overtake the prisoner?

66. Three men, named Adams, Brown, and Clark, with their sons, Daniel, Edward, and Francis, have each a piece of land in the form of a square. Mr. Adams's piece is 23 rods longer on each side than Edward's, and Mr. Brown's piece is 11 rods longer on each side than Daniel's. Each man has 63 square rods of land more than his son. Which of these persons are father and son respectively?

67. The stock of a factory is divided into 32 shares, and is owned equally by 8 persons. A sells 3 of his shares to a ninth person, and B sells 2 of his shares to the company. What part of the stock does A then own?

68. A merchant sold goods on a commission of 4 % ; having remitted the net proceeds to the owner, he was allowed $\frac{1}{4}$ % for prompt payment, which amounted to \$15.60. Find the amount of his commission.

69. Hiero, king of Syracuse, ordered a crown to be made of pure gold, but suspecting the workmen had debased it, he requested Archimedes, the celebrated mathematician, to detect the fraud. Archimedes found that a quantity of gold, and a quantity of silver, each of equal weight with the crown, displaced 13 ounces and 23 ounces of water, when immersed in it. The crown displaced 16 ounces of water. As the crown weighed 5 pounds, how many pounds of gold, and how many pounds of silver did the crown contain?

70. A refiner melts 14 pounds of gold 22 carats fine, with 16 pounds of gold 20 carats fine. How much alloy must be added to make the composition 18 carats fine?

71. A man bought 100 chairs for \$100, paying 50 cents apiece for the first lot, \$3 apiece for the second lot, and \$5 apiece for the third lot. How many of each kind did he buy?

72. There is an island 73 miles in circumference, and three men start from the same point at the same time, to walk around it, in the same direction. A walks 5 miles, B, 8 miles, and C 10 miles an hour. In what time will they all be together again, traveling 12 hours a day?

73. Two men mow a square meadow. One of them being a faster mower than the other agrees to take the outside swath and cut off all the corners. What part

of the whole piece will each mow, if there are 12 swaths on each?

74. How high must an object be to be seen at a distance of 40 miles at sea? And at what distance are the Andes visible at sea, their height being 4 miles?

75. One of two poles is just $\frac{3}{4}$ as long as the other, and they are set up in a vertical position, at a distance apart equal to the sum of their lengths. A point on the ground between them just $\frac{3}{7}$ the distance from the foot of the longer pole to the foot of the shorter pole is 30 feet from the top of each pole. Find the distance between the tops of the poles.

76. A church whose spire is 180 feet high stands on a plane. North of the spire, at a distance of 300 feet from its top, is a stake. Measuring east from this stake a distance of 400 feet, I find that I am just 80 feet from the foot of a column 15 feet high, which is in a direct line between me and the spire. Find the distance from the top of the column to the top of the spire.

77. A certain quantity of water runs into a coal-mine, in a given time. When full, the mine can be emptied in 60 hours by an engine of 12 horse-power; and in 80 hours by an engine of 10 horse-power. It will require an engine of what horse-power to empty it in 48 hours, and an engine of what horse-power to discharge the water as fast as it runs into the mine?

Ans. $\left\{ \begin{array}{l} 14 \text{ horse-power.} \\ 4 \text{ horse-power.} \end{array} \right.$

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